

## Objective System Analysis of Macroeconomic Systems

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A non-traditional criterion can be applied to the solution of the problem of the single matrix model choice by sorting-out many matrix models-candidates (the OSA algorithm - Objective System Analysis). Using the OSA algorithm on the sliding interval of time, we can evaluate the change of the diversity of a complex economic macrosystem. As an example, the intervals of more and less intensive development of the DDR economic system are shown.

### 1. Introduction

Constantly increasing consideration is being given to the modeling of macroeconomic systems of the regional, national and global scales during the last decades. A great variety of methods and models are used for this purpose (sometimes very complex and sophisticated) (LEONTYEFF, [2], DADAYAN, [1]). Main difficulties in constructing such models are related to the fact that they comprise a great quantity of variables but time series describing these variables are rather limited.

To solve many modeling and forecasting problems for complex (large-scale) dynamic systems using short samples, the Group Method of Data Handling (GMDH) developed in the V. M. Glushkov Institute of Cybernetics of the Ukrainian SSR Academy of Sciences has been successfully used during two last decades. The method and examples of its application for identification and forecasting of complex technical and environmental systems are described in many of books and articles (e. g. IVACHNENKO, A. G. et al. [4], IVACHNENKO, A. G., [3], IVACHNENKO, A. G., MULLER, J. A., [5], IVACHNENKO, A. G., STEPASHKO, V. S. [6], IVACHNENKO, A. G., YURACHKOYSKI, Yu. P., [7], FARLOW, S. E., [8], IVACHNENKO, A. G., KOZUBOVSKI, S. F., [9], etc. ).

The GMDH was also successfully used for modeling some laws governing the United Kingdom and German Democratic Republic economics. In this case, one of the GMDH algorithms was used - the Objective System Analysis (OSA) (IVACHNENKO, A. G. et al. [11], IVACHNENKO, A. G., [12]). As distinct from traditional GMDH algorithms (combinatorial, iterative, multiplicative ones) sorting out (sifting) vector models, in which a model corresponds to one equation, the sorting-out of matrix models is carried out in the OSA algorithms and the model is searched for in the form of a system of equations.

### 2. An Example: Analysis of GDR Economic System

The present report describe an application of the OSA algorithm for self-organization of forecasting models. As an example, the economic system of GDR is used. Models in the form of polynomials are synthesized - the analogs of differential equations.

The initial data array (Table.1.) comprises 25 variables:

Table 1. Initial Data for Modeling GDR Economics

Variables	Years																			
	1962	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981
$x_1$	1	0.749	0.421	0.431	0.497	0.508	0.284	0.310	0.375	0.445	0.340	0	0.121	0.217	0.271	0.282	0.314	0.315	0.314	0.301
$x_2$	0	0	0	0	0	0.005	0.132	0.197	0.197	0.197	0.295	0.394	0.390	0.339	0.387	0.354	0.413	0.581	0.781	1
$x_3$	0	0.012	0.023	0.047	0.085	0.120	0.160	0.200	0.250	0.297	0.342	0.398	0.449	0.527	0.602	0.070	0.762	0.854	0.934	1
$x_4$	0	0.102	0.245	0.254	0.254	0.254	0.373	0.373	0.373	0.373	0.462	0.699	0.715	0.750	0.794	0.847	0.902	0.947	0.980	1
$x_5$	0	0.063	0.125	0.188	0.250	0.418	0.583	0.750	0.875	1	0.938	0.875	0.730	0.573	0.468	0.375	0.313	0.310	0.303	0.300
$x_6$	0.344	0.435	0.595	0.763	0.779	0.840	0.870	0.931	0.931	0.931	0.939	0.946	1	0.939	0.847	0.824	0.695	0.527	0.374	0
$x_7$	0	0.027	0.058	0.094	0.135	0.179	0.222	0.208	0.318	0.371	0.422	0.469	0.521	0.579	0.650	0.720	0.779	0.839	0.920	1
$x_8$	1	0.955	0.909	0.804	0.773	0.082	0.636	0.545	0.500	0.455	0.409	0.304	0.318	0.273	0.182	0.136	0.091	0.045	0	0
$x_9$	1	0.903	0.840	0.719	0.700	0.770	0.821	0.845	0.847	0.833	0.787	0.702	0.579	0.419	0.262	0.138	0.053	0.012	0	0.005
$x_{10}$	0.267	0.092	0	0.010	0.006	0.054	0.058	0.150	0.242	0.212	0.242	0.393	0.007	0.840	0.830	0.834	0.998	1	0.709	0.850
$x_{11}$	0	0.019	0.074	0.059	0.105	0.138	0.138	0.192	0.260	0.295	0.375	0.401	0.575	0.587	0.717	0.781	0.857	0.895	0.930	1
$x_{12}$	0	0.013	0.039	0.075	0.123	0.157	0.220	0.278	0.332	0.380	0.442	0.500	0.558	0.619	0.679	0.741	0.806	0.809	0.933	1
$x_{13}$	0	0.020	0.045	0.084	0.127	0.170	0.215	0.205	0.319	0.370	0.420	0.475	0.544	0.620	0.684	0.752	0.825	0.885	0.932	1
$x_{14}$	0	0.013	0.042	0.077	0.115	0.158	0.201	0.247	0.295	0.349	0.399	0.461	0.527	0.608	0.669	0.730	0.816	0.874	0.925	1
$x_{15}$	0	0.020	0.050	0.071	0.091	0.149	0.190	0.221	0.256	0.287	0.370	0.459	0.488	0.570	0.672	0.754	0.816	0.870	0.936	1
$x_{16}$	0	0.085	0.130	0.180	0.254	0.492	0.542	0.271	0.644	1	0.780	0.678	0.814	0.780	0.627	0.763	0.780	0.508	0.271	0.508
$x_{17}$	0	0.080	0.172	0.259	0.345	0.448	0.534	0.003	0.655	0.724	0.759	0.810	0.879	0.931	0.966	0.966	0.983	0.983	1	1
$x_{18}$	0	0.023	0.041	0.059	0.082	0.110	0.142	0.183	0.228	0.279	0.338	0.397	0.461	0.525	0.594	0.667	0.744	0.820	0.909	1
$x_{19}$	0	0.003	0.021	0.056	0.101	0.132	0.189	0.249	0.307	0.357	0.413	0.481	0.563	0.638	0.690	0.757	0.834	0.890	0.924	1
$x_{20}$	0.072	0.015	0	0.039	0.071	0.086	0.104	0.173	0.239	0.289	0.313	0.374	0.532	0.644	0.721	0.789	0.861	0.909	0.957	1
$x_{21}$	1	0.949	0.881	0.814	0.746	0.508	0.458	0.729	0.356	0	0.220	0.322	0.220	0.373	0.237	0.220	0.220	0.492	0.729	0.492
$x_{22}$	0	0	0	0	0	0.200	0.400	0.000	0.800	1	1	1	1	1	1	0.800	0.600	0.400	0.200	0
$x_{23}$	1	0.920	0.853	0.779	0.700	0.647	0.588	0.529	0.471	0.412	0.358	0.304	0.250	0.196	0.147	0.118	0.088	0.059	0.029	0
$x_{24}$	0.002	0	0.013	0.024	0.036	0.047	0.060	0.071	0.084	0.094	0.120	0.144	0.167	0.309	0.476	0.571	0.691	0.790	0.900	1
$x_{25}$	0	0.010	0.017	0.027	0.034	0.044	0.051	0.061	0.069	0.076	0.086	0.128	0.170	0.390	0.594	0.661	0.730	0.790	0.914	1

,  $x_1$  - private consumption;  $x_2$  - social consumption;  $x_3$  - fixed capital of production sphere;  $x_4$  - production sphere investments;  $x_5$  - non-productive sphere investments;  $x_6$  - fixed capital quota;  $x_7$  - labor productivity (gross);  $x_8$  - fixed capital coverage;  $x_9$  - general population;  $x_{10}$  - total number of workers;  $x_{11}$  - workers of non-productive sphere;  $x_{12}$  - share of the growth of wage level in material sphere;  $x_{13}$  - gross national product;  $x_{14}$  - national production income;  $x_{15}$  - volume of foreign trade;  $x_{16}$  - share of accumulation;  $x_{17}$  - intensity of accumulation;  $x_{18}$  - share of the growth of pensions;  $x_{19}$  - real income;  $x_{20}$  - deposits at saving banks (savings);  $x_{21}$  - share of consumption;  $x_{22}$  - fixed capital quota - new assets;  $x_{23}$  - labor force;  $x_{24}$  - export prizes;  $x_{25}$  - import prizes.

For data normalizing, formula was used:

$$x_i^* = \frac{x_i - x_{i,\min}}{x_{i,\max} - x_{i,\min}},$$

where  $x_i^*$  is the normalized value of the  $i$ -th variable.

The results of the computing experiments using OSA algorithm are shown in Fig. 1. It is seen from the figure that the optimal complexity system of difference equations exists for each noise level. As in other GMDH algorithms, an increase in the noise level results in the shift of the minimum of the model selection criterion to the left, i.e. to the simplification of the system, to a reduction of the number of equations forming the system.

What is more, an astonishing experimental result resides in the fact that the algorithm practically never selects a system comprising more than five to six equations. This can be explained as follows: it is well known from the automatic control theory that time

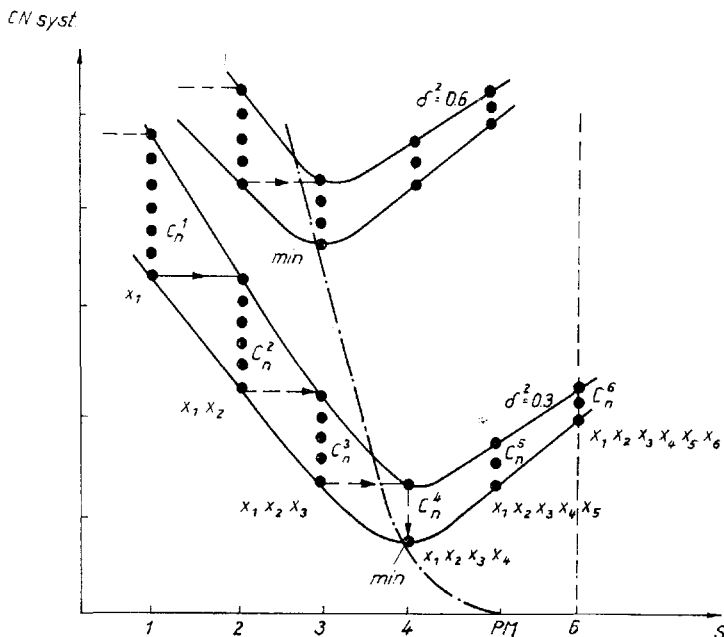


Fig. 1.  $CN_{\text{syst}}$  - system criterion,  $S$  - system complexity (number of equations in the system),  $PM$  - physical model,  $\delta^2$  - noise level

constants of individual feedback links are so different that the account of three to five time constants is sufficient ("A. Yu. Ishlinski's Rule"). Only an inexperienced engineer can describe a control system by the equation of the degree higher than the fifth one. Similarly, five different equations of the mentioned type take upon themselves the whole error variance. Computer can not play the role of the inexperienced engineer and generate too complicated system of equations comprising more than five-six difference equations of the described type.

In other words: the OSA algorithm finds sets of equations only for those variables whose behavior can be described by difference equations. The noncontradiction (minimum-of-bias) criterion (IVACHNENKO, A. G., [3]) is used in the OSA algorithm:

$$CN = \frac{\sum_1^N (\hat{q}_A - \hat{q}_B)^2}{\sum_1^N q_T^2} \rightarrow \min,$$

where  $q_A$  and  $q_B$  are the output variable values calculated from the models of the same structure whose coefficients are calculated using parts A and B of the initial data sample,  $q_T$  are the table values of the output variable.

### 3. The Results of Application of OSA Algorithm

Using the noncontradiction criterion, the algorithm has selected the following systems of equations: The best systems of the first and the second layer of selection:

$$\begin{aligned}x_{12(k)} &= 2.078 x_{12(k-1)} - 1.091 x_{12(k-2)} \\x_{7(k)} &= 2.083 x_{7(k-1)} - 1.168 x_{7(k-2)} + 0.067 x_{13(k-1)} \\x_{13(k)} &= 0.017 + 1.675 x_{13(k-1)} - 0.625 x_{13(k-2)}\end{aligned}$$

The best system of the third layer:

$$\begin{aligned}x_{7(k)} &= 0.032 + 1.067 x_{7(k-1)}, \\x_{13(k)} &= 1.903 x_{13(k-1)} - 1.140 x_{13(k-2)} + 0.248 x_{18(k)}, \\x_{18(k)} &= 1.347 x_{18(k-1)} - 0.427 x_{18(k-2)} + 0.224 x_{7(k)} - 0.182 x_{7(k-1)} \\&\quad + 0.085 x_{13(k-1)}.\end{aligned}$$

The minimum of the system criterion:  $CN_{\text{sys}} = 4.52 \cdot 10^{-5}$ . After the third layer, the minimum of the criterion begins increasing, therefore the sorting-out of the sets of equations terminates.

Step-by-step integration of the sets of equations has given the results presented in Table 2.

The models for other variables have been synthesized using the GMDH combinatorial algorithm COMBI with noncontradiction criterion. In this case, previous (delayed) values of the output variables and essential variables are given as arguments (initial complete polynomial was given and the COMBI algorithm has chosen only its essential terms):

$$\begin{aligned}x_{i(k)} &= a_0 + a_1 x_{i(k-1)} + a_2 x_{i(k-1)} + b_0 x_{7(k)} \\&\quad + b_1 x_{7(k-1)} + b_2 x_{7(k-2)} + c_0 x_{12(k)} + c_1 x_{12(k-1)} \\&\quad + c_2 x_{12(k-2)} + d_0 x_{13(k)} + d_1 x_{13(k-1)} + d_2 x_{13(k-2)} \\&\quad + l_0 x_{18(k)} + l_1 x_{18(k-1)} + l_2 x_{18(k-2)}.\end{aligned}$$

Table 2. Actual Data and Results of Forecasting of "Detailed" Variables by OSA

Variables	Years			Forecasting error	
	1979	1980	1981		
$x_7$	actual	0.839	0.920	1.000	$\delta^2 = 0.31$
	forecast	0.837	0.894	0.949	
$x_{12}$	actual	0.869	0.933	1.000	$\delta^2 = 0.13$
	forecast	0.866	0.921	0.968	
$x_{13}$	actual	0.885	0.932	1.000	$\delta^2 = 0.42$
	forecast	0.897	0.967	1.038	
$x_{18}$	actual	0.826	0.909	1.000	$\delta^2 = 0.108$
	forecast	0.833	0.933	1.032	

The forecasting accuracy (with forecasting interval of three years — 1979...1981) is as follows:

$$\begin{aligned}
 \delta_1^2 &= 281.15 & \delta_2^2 &= 15.64 & \delta_3^2 &= 0.71 & \delta_4^2 &= 695.06 \\
 \delta_5^2 &= 0.44 & \delta_6^2 &= 0.89 & \delta_8^2 &= 0.36 & \delta_9^2 &= 4802.88 \\
 \delta_{10}^2 &= 137.17 & \delta_{11}^2 &= 432.20 & \delta_{14}^2 &= 0.59 & \delta_{15}^2 &= 42.43 \\
 \delta_{16}^2 &= 71.57 & \delta_{17}^2 &= 374.71 & \delta_{19}^2 &= 3.93 & \delta_{20}^2 &= 102.90 \\
 \delta_{21}^2 &= 40.71 & \delta_{22}^2 &= 0.01 & \delta_{23}^2 &= 0.36 & \delta_{24}^2 &= 63.45 \\
 \delta_{25}^2 &= 15.30.
 \end{aligned}$$

It may be concluded that only 10 of 25 variables are forecasted by means of the OSA algorithm satisfactorily and good ( $\delta^2 < 1.0$ ). As is generally known, with  $\delta^2 < 0.5$  the forecast can be considered to be good and with  $\delta^2 < 0.8$  - satisfactory. With  $\delta^2 > 1.0$  the forecast bears misinformation.

#### 4. Qualitative (Fuzzy) Forecasting by OSA Algorithm Obtained with Shift of "Sliding Window" (Observation Interval)

The ordinate of the minimum of the noncontradiction criterion graph is an important index of the condition of the object being modelled which reflects the generalized level of its development (the complexity or variety level).

Applying the OSA algorithm for three basic intervals (1962-69, 1970-75, 1976-81), we can obtain a fuzzy qualitative conclusion on the trend of development of some economic system. The system perfects itself when  $S_{opt}$  increases from cluster to cluster. In the present case, we have obtained

$$\begin{aligned}
 S_{opt}(1962-69) &= 2, \\
 S_{opt}(1970-75) &= 3, \\
 S_{opt}(1976-81) &= 3,
 \end{aligned}$$

i.e., the GDR economic system has changed to a stable level of development. The value of  $S_{opt}$  is equal to the number of equations in the optimal forecasting system found by means of OSA.

## References

- [1] DADAYAN, V. S.: Global economic models. Nauka, Moscow 1971 (in Russian)
- [2] LEONTYEFF, V. V.: Research into the structure of American economics. Gosstandartizdat, 1958 (in Russian)
- [3] IVACHNENKO, A. G.: Inductive method of self-organization of complex system models. Naukova Dumka, Kiev 1983 (in Russian)
- [4] IVACHNENKO, A. G.; KOPPA, Yu. V.; STEPASHKO, V. S. et al.: Handbook of standard modeling programs. Technika, Kiev 1980 (in Russian)
- [5] IVACHNENKO, A. G.; MÜLLER, J. A.: Selbstorganization von Vorhersagemodellen. VEB Verlag Technik, Berlin 1984
- [6] IVACHNENKO, A. G.; STEPASHKO, V. S.: Noise immunity in modeling. Naukova Dumka, Kiev 1985 (in Russian)
- [7] IVACHNENKO, A. G.; YURACHKOVSKI, Yu. P.: Modeling of complex systems by experimental data. Radio i Svyaz, Moscow 1987 (in Russian)
- [8] FARLOW, S. E. (ed.): Self-Organizing Methods in Modeling. GMDH Algorithms. Marcel Decker Inc., New York-Basel 1983
- [9] IVACHNENKO, A. G.; KOZUBOVSKI, S. F.: Rechnergestützte Selbstorganization Mathematischer Modelle (GMDH - Group Method of Data Handling), 2. Duisburger Kolloquium Automation und Robotik, Universität Duisburg, 1987, S. 30
- [10] IVACHNENKO, A. G.; Kostenko, Yu. V.: System analysis and long-range prediction of quasi-static systems based on model self-organization. Part 1: System analysis on the level of trends. Avtomatica (1982) 3, 11-19, (in Russian)
- [11] IVACHNENKO, A. G.; KOSTENKO, Yu. V.; GOLEUSOV, I. V.: System analysis and long-range prediction of quasi-static systems based on model self-organization. Part 2: Objective system analysis without a priori setting of external disturbances. Avtomatica (1983) 3, 3-11 (in Russian)
- [12] IVACHNENKO, A. G.: Sorting-out methods of modeling and clusterization (A review of work on GMDH in 1983-88). Avtomatica (1988) 4, 3-16 (in Russian)

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