PREDICTED COSTS AND PERFORMANCE EXPECTATIONS: QUESTIONING ASSUMPTIONS OF LINEARITY WITH GMDH NEURAL NETWORKS

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Proposed Running Head: Predicting Educational Costs with GMDH Neural Networks

Abstract.
Determining the cost of education continues to one of the more elusive ongoing tasks in educational and economic research. Our efforts to tackle this objective are often plagued by difficulties in determining the extent to which educational spending rationally relates to educational costs in a system that is political by design, diminishing our capacity to use traditional economic methods for price setting. One truth that is beginning to emerge from the literature is that the cost of education is not constant. Education costs vary widely across institutions by region, by the organizational and structural characteristics of schooling and by the needs of individual students. A question being addressed more recently, is whether the cost of education varies by performance outcomes or expected performance outcomes of schools. That is, is there a cost - quality relationship that can rationally be used in price setting for public education? This study applies an alternate methodology, a flexible non-linear method known as Group Method of Data Handling to test the sensitivity of cost to changing performance expectations. In addition, this study tests the sensitivity of costs to changes in student population characteristics, holding performance expectations constant, challenging our current assumption, as manifested in policy designs, that costs of additional students with perceived special needs are linear and incremental. In general, the findings support that higher levels of minimum adequacy are required than currently exist for Texas school districts, and that this pattern generally ascribes to diminishing marginal costs. In addition, the findings do suggest that in some cases, and at some levels, changes in student population characteristics may result in comparatively explosive costs.

JEL Classification:

Key Words: Cost Function; Neural Networks

1. Introduction

Determining the cost of education continues to one of the more elusive ongoing tasks in educational and economic research. Our efforts to tackle this objective are often plagued by difficulties in determining the extent to which educational spending rationally relates to educational costs in a system that is political by design, diminishing our capacity to use traditional economic methods for price setting. One truth that is beginning to emerge from the literature is that the cost of education is not constant. Education costs vary widely across institutions by region (Chambers, 1995; McMahon, 1994), by the organizational and structural characteristics of schooling (Chambers, 1995) and by the needs of individual students (Chambers, Parrish and Hikido, 1996). A question being addressed more recently, is whether the cost of education varies by performance outcomes or expected performance outcomes of schools (Imazeki and Reschovsky, 1998; Duncombe, Ruggiero, and Yinger, 1996; Duncombe and Miner, 1996; Alexander, Augenblick, Driscoll, Guthrie and Levin, 1995). That is, is there a cost - quality relationship that can rationally be used in price setting for public education? Odden and Clune (1998) recently suggested the following as a crucial endeavor toward achieving educational adequacy:

> *"The medium to long-term goal should be to set spending at a level that would allow the average school to teach the average student to rigorous state or district performance standards."* (Odden and Clune, p. 164)

Unfortunately, there is little uniformity to our understanding of the sensitivity of student performance to per pupil spending (Hanushek, 1997, 1996, 1994, 1989; Hedges and Greenwald, 1996; Hedges, Laine and Greenwald, 1994), or the inverse of per pupil expected costs to changing performance demands (Duncombe, Ruggiero and Yinger, 1996; Duncombe and Miner, 1996). Despite continued lack of agreement on research findings, some standards for the assessment of the relationship seem to be emerging from the literature. For one, there is growing consensus that such relationships should be analyzed at the student, or school level, rather than district level (Harter, 1999). In addition, it has become generally accepted that the resources of interest are typically not total, but instructional expenditures per pupil (Cooper et al., 1994; Dolan and Schmidt, 1987), though other methods of disaggregation of spending have also proven useful (Harter, 1999; Brewer, 1996). In addition, the relative complexity of the structure of schooling, and potential for both interactive and non-linear relationships among variables warrants the consideration of a variety of alternate techniques (Baker, 1999). Figlio (1999) also points to the value of more flexible non-linear models including a translog approach for estimating the magnitude of the input-output relationship.

1.1 The Policy Context

This study is primarily a methodological exercise but is performed within the policy context of the state of Texas. While prior analyses have been done with GMDH on Vermont data (Baker 2000, 1999) the state of Texas provides a variety of opportunities with respect to validating the usefulness of GMDH. For one, Texas, in recent years has been a leader in the development of a comprehensive statewide indicator system including demographic, financial and student performance data available in electronic form at the district and the school level. As a result, Texas data has been studied extensively including both production analyses (Harter, 1999) and cost analyses (Reschovsky and Imazeki, 1999). To a large extent, this study relies on the cost function framework set forth by Reschovsky and Imazeki and applied to earlier years of Texas data. The GMDH method to be tested in this study, a complex pattern learning algorithm, is also presumed to benefit from having substantially large sample sizes. Texas 1,062 school districts provide a unique opportunity in this respect.

1.2 Neural Networks and Predictive Modeling

The primary objective of neural networks is predictive modeling. That is, the accurate prediction of non-sample data using models estimated to sample data. With cross-sectional data, this typically means the accurate prediction of outcome measures (dependent variable) of one data set generated by a given process, by providing input measures (independent variables) to a network (deterministic non-linear regression equation) trained (estimated) to a separate data set generated by the same process. With time-series data, the objective is typically forecasting, given a sample set of historical time-series realizations. This is a departure from traditional econometric modeling where a theoretically appropriate model is specified then estimated using the full sample for purposes of hypothesis testing, the primary objective being inference.

Identification of the best predicting model typically begins with subdividing the sample data set into two components, the *Training Set* and the *Test Set*, a hypothetical set of non-sample data extracted from the sample, against which prediction accuracy of preliminary models can be tested. Typically, the *test set* consists of up to 20% of the sample (*Neuroshell 2* User's Manual (WSG), 1995, p. 101). The objective is to identify the model which, when estimated to the training set, most accurately predicts the outcome measures of the test set as measured by absolute error or prediction squared error. It is then expected that the same model will best predict non-sample data, sometimes referred to as the *production set* (WSG, 1995, p. 101).

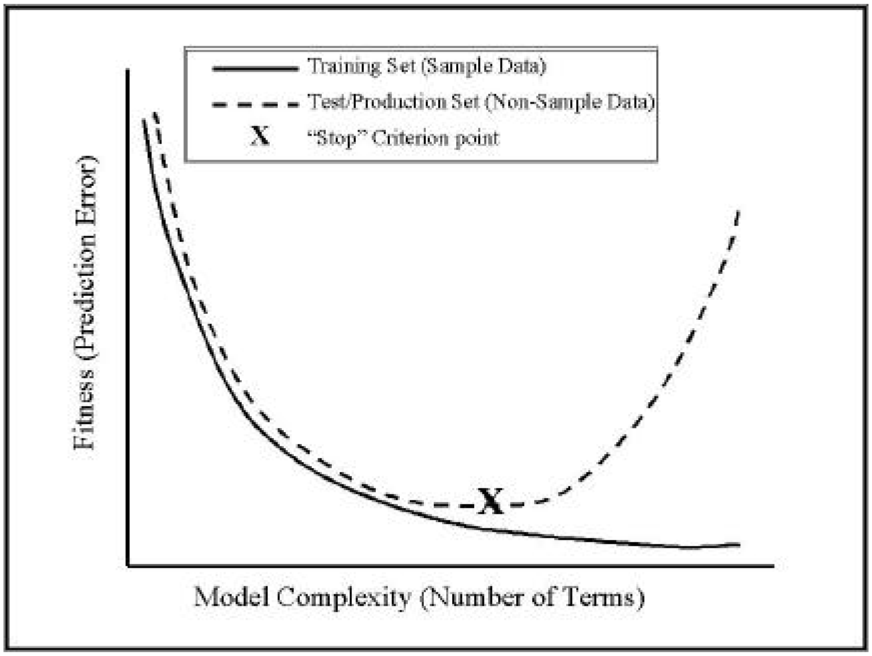
Two methods are commonly used for estimating the deterministic neural network model: (1) iterative convergent *learning* algorithms and (2) genetic algorithms. Superficially, the iterative, convergent method begins by randomly applying a matrix of coefficients (connection weights) to the relationships from each independent variable to the dependent variable of the training set. The weights are then used to predict the outcome measure of the test set. Prediction error is assessed, and either a new set of random weights are generated, or learning rate and momentum terms dictate the network to incrementally adjust the weights based on the direction of the error term from the previous iteration (WSG, 1995, pp. 8, 52, 119). The process continues until several iterations pass without further improvement of test set error.

The genetic algorithm approach begins by randomly generating pools of equations. Again superficially explained, initial equations are estimated to the training set and prediction accuracy of the outcome measure is assessed using the test set to identify a pool of the "most fit" equations. These equations are then hybridized or randomly recombined to create the next generation of equations. That is, parameters from the surviving population of equations may be combined, or excluded to form new equations as if they were genetic traits. This process, like the iterative, convergent application of weights continues until no further improvement in predicting the outcome measure of the test set can be achieved.

A common concern regarding flexible non-linear models is the tendency to "overfit" sample data (Murphy, Fogler and Kohler, 1994). It has been shown, however, that while iterative or genetic, selective methods can generate complex non-linear equations that asymptotically fit the training set, the prediction error curve with respect to non-linear complexity for the test set is **U** (Murphy, Fogler and Kohler, 1994) or **V** (Farlow, 1984) shaped;

that is, beyond an identifiable point, additional complexity erodes, rather than improves, prediction accuracy of the test set (See Figure 1).

Figure 1. Model Optimization for Neural Networks



1.3 Group Method of Data Handling (GMDH)

A.G. Ivakhnenko (1966)[1] proposed an algorithm called Group Method of Data Handling (GMDH) for identifying a best predicting polynomial equation.[2] More recently, the problem of estimating GMDH polynomials has been addressed with genetic algorithms and neural network methods (Madala and Ivakhnenko, 1994; WSG, 1995). GMDH polynomial fitting differs from other types of neural networks in that estimation does not, by necessity, involve extracting a test set. While true, inductive GMDH neural networks do involve *test set* extraction, *Neuroshell 2*, a software package commonly used in financial analysis, and used in this study, employs a selection criterion referred to as FCPSE (Full Complexity Prediction Squared Error) to estimate

[1] In Farlow, 1984

[2] via a Kolmogorov-Gabor specification. That is,

$y = a_0 + \sum_{i=1}^{M} a_i x_i + \sum_{i=1}^{M}\sum_{j=1}^{M} a_{ij} x_i x_j + \sum_{i=1}^{M}\sum_{j=1}^{M}\sum_{k=1}^{M} a_{ijk} x_i x_j x_k$, where $X(x_1, x_2, \ldots x_m)$ is the vector of inputs and $A(a_1, a_2, \ldots a_m)$ is the vector of coefficients or weights (Liao, 1992).

the optimal predicting polynomial using the full sample. FCPSE consists of a combination of Training Squared Error[3], an overfitting penalty and additional penalty measures for model complexity.[4]

Previous studies have implicated GMDH over other neural networks and over conventional regression methods as a superior predictor, by measure of Mean Absolute Percentage Error (MAPE) on non-sample data, of student performance (Baker, 1999) and of educational spending (Baker and Richards, 2000). Other researchers have implicated GMDH for it's usefulness in social science due to its ability to identify non-linear relationships and interactions and present those relationships in the form of a deterministic regression equation, a more interpretable outcome than more black-box methods such as backpropagation (Liao, 1992).

1.4 Sensitivity Analysis and Simulation

Sensitivity analysis is a useful method for characterizing the response of a dependent variable to changes or differences in levels in individual or multiple input measures. Sensitivity analysis is particularly useful for drawing inferences from complex, interconnected models, such as structural models (Kaplan and Elliott, 1997), systems dynamics models (Richmond and Peterson, 1997), or neural network algorithms that flexibly derive complex non-linear forms. The difficulty with traditional interpretation of these models, that is, analysis of coefficients, is that the collective effects of the various linear and non-linear relationships often yield counterintuitive results regarding changes in the outcome measure with respect to changes in inputs.

Neural networks and other complex, interconnected models are particularly useful for performing sensitivity analyses because they often provide greater prediction accuracy than their more conventional counterparts (Baker, 1999; Baker and Richards, 2000). Greater prediction accuracy implies greater sensitivity of outcomes to changes in inputs. Greater prediction accuracy of models trained on one set of data, but applied to another set of data, implies that the sensitivities derived from the first set of data are generalizable, and useful for making inferences regarding the nature of the relationship between inputs and outcomes.

[3] Referred to as Norm.MSE. Discussed in more detail in WSG (1995) pp. 149-151.

[4] WSG retains proprietary rights to the design of FCPSE and therefore does not disclose the formula for its determination (WSG, 1995, p. 150)

While not commonplace in educational research or social science research in general, neural network sensitivity analysis is used with increasing regularity in medicine (Casciani and Parham, 1998; Parham and Casciani, 1998; Reid, Nair, Kashani and Rao, 1994) and in engineering and operations research (Sharda and Wang, 1996). Although neural network sensitivity analysis has not yet been applied in educational policy research, Kaplan and Elliott (1997) provide an example of complex sensitivity simulation in educational policy research using Structural Equation Modeling.

The basic difference between the Neural Network approach to sensitivity analysis and that presented by Kaplan and Elliott, is that the Neural Network approach allows the computer to select a best predicting model given the data available and is therefore inductive rather than deductive. While the structural model, by way of inductive exploration and deductive testing, may appear both theoretically reasonable and statistically acceptable for representing the system in question, it may or may not be adequately sensitive. That is, it may not capture each of the possible interconnected relationships in the data or the full extent of non-linearity in the relationships.

1.5 Goals of the Study

The goal of this study is, within a relatively simple and established cost function framework, to test the potential usefulness of flexible GMDH models and sensitivity analysis for generating cost estimates for school districts of varying student populations with the goal of achieving performance standards. As well as focusing directly on the sensitivity of cost to performance expectations, the simulations that follow also assess the sensitivity of cost to changing demographic features of the student population while holding performance expectations constant. Essentially, these simulations ask the question, how do changes in exogenous factors affect our costs of achieving high performance. Many states, including Texas make efforts to compensate districts for the expected extra costs of serving economically disadvantaged or limited English proficient children, but generally these additional funds are not tied to any expected level of performance outcome. Contrary to the standard linear weighting system design employed by most states in compensating these costs, it is expected that GMDH will reveal turning points and/or critical thresholds at which marginal costs may either become diminished, or perhaps even explosive.

2. Methods

2.1 Data

While efforts in recent production function analyses have been centered on using the pupil or school as the unit of analysis, cost function studies have continued to focus on the district as the unit of analyses. For educational production, it is assumed that most of the learning processes occur at the school and pupil level and that the bulk of educational interventions, and variance among interventions occur at the level of schools and classrooms. Yet, in terms of cost or spending, local revenue generating or state and federal revenue allocation, the district remains the basic functional unit. Therefore, district level data on Texas school districts were used.

Most data were acquired from the Texas Education Agency web site, drawn from the Texas "Snapshot" file series and supplementary tables of student population background characteristics and district financial tables.[5] Of particular importance are the student performance outcome measures and spending measures. Regarding student outcomes, this study takes a *Value Added* approach. That is, we regress spending on current levels of student performance, as measured by the Texas statewide assessments (TAAS), given prior performance of the same students. Particular data used include a TAAS composite score (average across curricular content areas) for 7th and 8th graders in 1997-98, and 4th and 5th graders in 1994 - 95. School average ACT scores and measures of participation rates were also used. Regarding spending measures, in one set of models total expenditures per pupil are used while in another the focus is placed on instructional expenditures per pupil. Variables regarding the nature of the student population that were expected to reflect cost differences (or at least spending differences as generated by the current Texas formula) include (1) percent of students enrolled in high school (2) percent economically disadvantaged (3) percent receiving special education services.[6]

Development of a cost model for education requires data on the differential costs of schooling inputs. A major source of exogenous differential costs of instruction are expected to be the differential costs of teaching personnel. Chambers (1995) and McMahon (1994) have both addressed the issue of differential teacher costs and created indexes for accounting for cost

[5] Available at http://www.tea.state.tx.us/perfreport

[6] This study does not explore this issue to the same extent as that of Reschovsky and Imazeki (1999) as the intent of this study is to explore alternative methodologies rather than derive definitive cost indexes. Reschovsky and Imazeki include a separate classification for severely disabled students.

adjustments. This study uses Chambers' Teacher Cost Index for Texas school districts to account for the differential cost of teachers as inputs.[7]

2.2 Theoretical Issues Concerning the Education Cost Function

The education cost function takes the basic form:

(1) $E_u = f(X_u, P_u, \varepsilon_u)$

Where per pupil expenditures, E_u, are specified as a function of school inputs, X, a vector of input prices, P, and a vector of unobserved characteristics, ε. Given the available data, we can express a simple linear regression form of the cost function as:

(2) $ln\text{PPE} = \beta_0 + \beta_1 ln\text{Price}_i + \beta_2\text{Students}_i + \beta_3 ln\text{Output}_i + \beta_4 ln\text{Scale}_i + u_i$

Where price or input prices is represented by Chambers' (1995) TCI for each district *i*, students is represented by a matrix of student characteristics (%high school, %special ed., %gifted and talented, %economically disadvantaged and %limited English proficient) and output is the value added measure of TAAS generated by included current and lagged forms of the variable. An additional measure, scale, is added to compensate for cost differences that are a result of diseconomies of scale. A substantial volume of literature has addressed the issue of dramatically higher costs of very small school districts. As a measure of size, we include district enrollment. In keeping with prior studies, including that of Reschovsky and Imazeki, we also include the square of district enrollment to represent the assumed non-linearity of this relationship. In keeping with the expectation of diminishing returns, or in this case diminishing marginal costs with respect to increased performance outcomes we accept the log-log form of the input-outcome relationship.

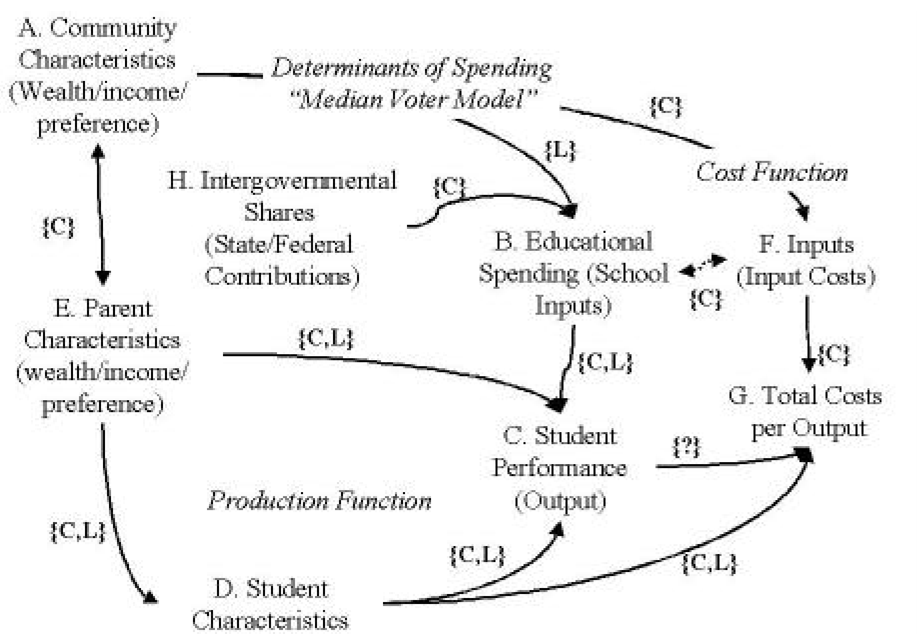
A problem stimulating greater interest and further development of cost and production models in education in recent years is that pupil outcomes are expected to be endogenous with respect to costs. That is, while it is expected that higher performance levels do indeed demand

[7] TCI, Teacher Cost Index may be downloaded from The American Institutes for Research at www.air-dc.org/

higher unit costs, it is also assumed that higher existing performance levels may in fact be depended on prior levels of spending (inferred to be equivalent to cost in such models). Therefore, it becomes particularly difficult to discern whether higher performing students require higher costs, or whether they are performing at higher levels because their schools/communities have more available resources. A measurable symptom of endogenous inputs is where some of the inputs, X, are correlated with the error term of the estimated model. Under these circumstances the coefficients for the inputs are assumed biased.

Figure 2 displays a fairly complex, systems diagram linking the standard production and cost model structures and adding to them the standard model for linking community characteristics to school district spending behavior. In brief, spending model is based on community and parent characteristics (A&E, usually the characteristics of the "median voter") and intergovernmental contributions (state and federal general and categorical aid) serving as predictors of educational spending (B). Spending, in turn, becomes one of the potential but still debatable predictors of student outcomes along with the characteristics of the student and parent population (D&E). A more difficult aspect of the model is distilling costs from spending, the represented structure certainly being open for debate. We can presume, as we have noted in our cost function, that total costs are a function of input costs (F), output levels (C) and student characteristics (D).

Figure 2. Systems model of spending, production and cost in education



One approach to filtering out the effects of endogenous student outcomes on cost is to use a simultaneous equation or instrumental variable (IV) approach such as two-stage least squares (2SLS). By the representation in Figure 2, it would seem appropriate that student performance variables be endogenously estimated as a function of schooling inputs (B), parent characteristics (E) and student characteristics (D), and then the fitted values of the performance outcomes used in the estimation of per pupil costs (G). Reschovsky and Imazeki (1999) bypass (B), using A, E, D and H to predict C. Conceptually, they chose to use a common set of predictors of discretionary local spending (A) to represent the spending input itself (B), less the intergovernmental contributions (H).[8]

The other difficulty with conceptualizing and subsequently estimating such a model is appropriately accounting for the time dimensions of the system. Education cost-functions are generally estimated with the data that are available for the years that are available. The {} notations in Figure 2 refer to expected lags of individual relationships in the system. Following one pathway through the system we might expect costs {t} (at time "t"), to be contemporaneously and/or lagged relationship with endogenous performance outcomes {t-1}, which in turn have a lagged response to spending {t-2…}, that itself has a lagged response the measures of voter capacity to spend {t-3…}. While the nature and extent of each of these lagged structures are debatable, their existence and cumulative effect through the system is generally supported. This issue is raised because often in IV approaches, we find ourselves using current values of instrumental variables to predict past values of endogenous variables. Conceptually, this approach is deeply flawed, yet it often produces reasonable results simply because the various instruments are relatively stable over time, such that current values serve as reasonable proxies for past values in a cross-sectional analysis.

[8] While conceptually awkward, the choice to substitute more purely exogenous instruments (determinants of spending) in place of a lagged endogenous spending variable is consistent with the standard recommendation in IV approaches that the instruments not be highly correlated with the error term (See Pindyck & Rubenfeld, 1981, p. 329). Where Y_t is a dependent variable, and Y_{t-1} is used as an instrument we can expect that Y_t and Y_{t-1} display serial correlation and as a result there is a high likelihood that Y_{t-1} will display correlation with the error term of prediction of Y_t. However, where variables so highly correlated with Y_t (in this case per pupil spending) such as income and property wealth are used, similar expectations of correlation between instruments and model error exist. In the end, use of lagged spending (A in our diagram) as an instrument produces consistent results with use of determinants of spending (H, A , E &D in our diagram) as instruments in a 2SLS cost-function specification.

In this study, in our preliminary analyses in estimating educational costs as a linear function, we apply both OLS estimation and 2SLS estimation. For our 2SLS estimation the ultimate objective is the prediction of 1997-98 values of educational costs (G). Thus we regress 1997-98 costs on 1997-98 performance (TAAS Composite, ACT), lagged 1994-95 performance (TAAS Composite), a matrix of 1997-98 student characteristics (D) and the cost of teacher inputs (F). The endogenous outcome measures are regressed on a matrix of 1994-95 instruments including spending inputs (B) and student and parent characteristics (D&E).

The comparison of OLS and 2SLS models is performed to point to a potential shortcoming of the GMDH analyses that follow. That is, potential underestimation of the magnitude of the sensitivity of cost to performance. Each of the GMDH cost functions estimated represent only non-linear forms of the respective OLS equations. It seems reasonable that indirect or two-stage least squares methods might be simulated by generating sets of predicted values for endogenous variables using neural networks trained on sets of predetermined instruments and that those predicted values could be used in subsequent training of second stage neural networks. An alternative would be to apply the new insights on potential functional forms generated by single stage neural networks as parameters to be tested in the context of a more conventional simultaneous equation model. Attempts are ongoing to validate the statistical properties of the outcomes of the 2-stage neural network approach and the relative usefulness of each of the two alternatives presented. It should be noted, however, that while 2SLS estimation may be more theoretically appropriate than OLS for the function in question, none of the inputs in question formally display statistical properties of endogenous variables in OLS estimation - that is, correlation between performance measures and residuals of the cost functions.

While the objective of later studies may be to better flush out the issues of time dynamics and endogenous features in the production-cost-spending model, the objective herein is to question and test the nature of the relationships represented by the individual connections between variables. That is, can we really expect variables like the cost of education to respond incrementally, uniformly and linearly to all other variables in the model or are these connections potentially more complex? Recall that even the standard cost model applied is assumed to benefit from inclusion of (1) a squared representation of the measure of diseconomies of scale and (2) log-log representation of the input-outcome "diminishing" returns relationship.

2.3 Measuring Model Fitness

Estimated GMDH models and OLS models are first compared for their ability to accurately predict the cost measures (PPE and INPP) of non-sample (N=100) districts. The comparison measures used are non-sample R-square, Mean Absolute Percentage Error (MAPE) and the standard deviation of the MAPE.

2.4 Predicted Costs and Performance Expectations

Two performance - cost sensitivity analyses were performed in an effort to understand the responsiveness of cost (total and instructional) to different levels of performance outcomes. In each case, the levels of current year TAAS performance were adjusted for production set schools (N = 100) changing the desired *Value Added.* In the first set of simulations, value added was incrementally increased across all districts in the production set from 10% performance increase to 50% performance increase, but capping increases at the maximum possible performance levels (TAAS(C) = 100). Thus, all schools in the simulation achieve either positive or "0" value added change at each level of the simulation. With a linear model, where the coefficient for TAAS is positive, such a simulation would simply yield positive incremental (or "0") cost changes across all districts. Where additional higher order terms and two and/or three-way interactions are included this may or may not be the case.

The second sensitivity analysis plays off of the relatively popular policy objective of finding better ways to determine adequacy levels of spending to yield high performance, or determine the cost of uniformly adequate or high performance. Thus, this simulation involves moving all districts to standard performance levels, beginning with median performance and moving to the current 75%ile and 95%ile levels. Note that this approach does result in reducing the performance level of 50%, 25% and 5% of production set schools respectively at each level of the simulation. The question of importance, however, is how such performance demands affect the levels of average and minimum costs.

2.5 Sensitivity of Costs to Changes in Exogenous Factors

The final simulation couples performance standards with cost differences yielded by exogenous characteristics of the student population. Manipulation of exogenous factors in such sensitivity analyses is perhaps most likely to produce reliable results as the estimated coefficients, linear or non-linear, are less likely to be biased. As noted earlier, Texas, like may other states provides supplementary funding for pupils classified in various ways including limited English proficiency (LEP) and economic disadvantage (as measured by eligibility for federal free and reduced lunch programs). Texas, like other states uses uniform, linear pupil weighting systems for distributing this additional aid, which results in a linear response of cost to numbers of children classified.

The exogenous sensitivity analyses include manipulation of (1) proportions of students identified as limited English proficient (from 0% to 70%) and (2) proportions of economically disadvantaged students (from 0% to 100%), in each case holding performance as measured by current year TAAS values constant at the median level. Assuming that the Texas weighting system is appropriate, we would expect each simulation to result in a linear (constant slope) response pattern where the slope reflects the additional funding provided by the state. Where GMDH yields lesser slopes than would be expected, the inference is that only some of the additional funds provided are being reflected in cost differences related to the funding criteria. In other words, where GMDH yields lesser slopes, the state is over-compensating districts. Thus, the converse, where GMDH suggest greater marginal increases for additional classified students than would be expected, Texas is likely under-compensating districts.

The potential value of using GMDH estimated models for this simulation is to identify where the patterns of cost response are not linear. For example, are there ranges for which dramatically increased predicted costs emerge from marginal increases in classified pupils, while for other ranges of marginal increases in classified pupils, negligible cost differences occur? Generally, we do not account for such patterns in state funding formulae, but the likelihood of such patterns are immanent.

3. Results

3.1 Estimation of Alternative Linear Cost Functions

Table 2 displays the results of the alternative linear cost functions, OLS and 2SLS for both total expenditures per pupil and instructional expenditures per pupil. The key differences between the OLS and 2SLS results is that the 2SLS approach helps significantly in filtering out the effects of outcome measures on cost after controlling for the effects of spending and other exogenous factors on outcomes. As a result, in both 2SLS models, the TAAS and lagged TAAS variables emerge significant.[9] This outcome may suggest some bias in the input-outcome sensitivities established with the single stage GMDH models that follow. In either case, however, OLS or 2SLS the sensitivity of cost to performance expectations is represented by a single linear coefficient that applies by definition (1) uniformly at all levels of inputs and outcomes and (2) uniformly across all districts. The same can be said of the coefficients that represent the sensitivity of cost to various exogenous factors including proportions of students who are limited English proficient and proportions of students who are economically disadvantaged. It is this result, in particular, that is called into question with GMDH.

3.2 Prediction Accuracy of OLS and GMDH

Prediction accuracy of GMDH with the district level cost data in this study proved less effective than in previous studies (Baker, 2000; Baker, 1999; Baker and Richards, 1999). Table 3 displays the measures of training and production set fitness and prediction accuracy (estimated GMDH equations are shown in Appendix A). For predicting total expenditures per pupil, GMDH slightly, but significantly outperforms OLS in terms of Mean Absolute Percentage Error of prediction. In addition, GMDH displays slightly greater fit over OLS to both training set (sample) and production set (non-sample) data. Regarding prediction of instructional expenditures per pupil, differences between OLS and GMDH are negligible, suggesting that the linear representation of all relationships in the model may be equally appropriate to the GMDH non-linear representation. It should be noted, however, that GMDH appears to have accomplished somewhat greater fit to the training set data.

3.3 Performance Expectation Simulations

Table 4 and Table 5 display the results of (1) the simulation of incremental improvements in performance to all districts and (2) the standards-based simulation of holding all districts constant at various levels of performance. In GMDH simulations, due to the presence of second and third order terms and two and three way interactions, linear changes in a given input will likely yield vastly different changes in outputs across cases, as well as different responses at different levels of the inputs and outcomes. The response will depend on the initial level of

[9] Where the TAAS and lagged TAAS variables of the second stage regression are actually the predicted values of these variables generated by the first stage regression.

inputs and outcomes, as well as the mix of other exogenous factors in the model. As a result it is common that the maximum spending district would be "leveled down" even where performance is incrementally increased. Similarly, a case often exists at the lowest extreme where a relatively high performing, low spending school or district exists, with a "desirable" mix of exogenous inputs such that that district can/will be leveled down to an even lower and seemingly unreasonable level of cost. Thus, the more pertinent comparisons in Table 4 and Table 5 for example, are of mean spending levels and 95th and 5th percentile levels of spending, where extreme responses have been truncated.

In table 4, while both the 95th percentile and mean are leveled down, the 5th percentile spending level remains relatively constant from the initial predicted value. Most importantly, the 5th percentile spending predicted values all substantially exceed the 5th percentile actual spending value, suggesting that while average and high spending districts surpass a threshold of adequacy for spending, it appears that low spending districts do not currently surpass the adequacy threshold for meeting increased performance demands. The implications of Table 5 are quite similar, pointing to higher levels of predicted cost for 5th percentile schools and in the case of trying to bring those schools from their current performance to 95th percentile performance levels additional incremental changes in cost are required. In general the data seem to suggest a reasonable range of total expenditures per pupil to be from about $5,100 (versus $4,622) to about $7,500 (versus $8,198) with a mean at about $5,800 or $5,900 (in keeping with the current $5,862).

The patterns of response are quite similar for the instructional expenditure simulations displayed in Table 6 and Table 7. Again both extremes continue to diverge in somewhat unreasonable patterns and the means remain relatively unchanged from actual means. In these simulations, GMDH predicts gradual increases in instructional costs for 95th percentile districts with increasing performance demands. In the case of 5th percentile expenditures, there appears to be a point at which additional costs are not incurred with further increases in performance demands. Predicted costs decline beyond a uniform 20% increase in performance demands across all districts (Table 6) or beyond raising all districts to the 75th percentile level of performance. In each case, 5th percentile predicted costs (approximately $2,700) again consistently exceed 5th percentile actual costs ($2,484) as well as the current (1998-99) Texas basic per pupil allotment ($2,396). In general, tables 6 and 7 seem to suggest a reasonable range of instructional expenditures for meeting high performance requirements to be from a low of around $2,700 (versus $2,484) to a high of around $4,400 (versus $4,267) with a mean of around $3,150.

3.4 Sensitivity to Exogenous Factors

Table 8 and Chart 1 display the sensitivity of cost to changing proportions of LEP students in a district, holding performance outcomes (TAAS) constant at the median. Thus, in theory, the simulation represents the cost of achieving median performance in each district at different levels of the exogenous input. Recall for one, that the sensitivity of cost to changing levels of LEP was represented as a simple linear coefficient in the OLS and 2SLS models. In addition, the Texas funding formula like many states provides a linear weighting supplement of 10% for each additional LEP classified child. For this simulation, we chose to model only the sensitivity of instructional costs. By nature of the funding allocation system, assuming the allocation of 10% meets, but does not exceed required levels, we would expect a simple linear response of cost (spending) replicating the 10% per LEP student allocation formula. Results of the simulation, however, do not meet this expectation, displaying negligible increase in predicted instructional costs per pupil across much of the range (from 0%LEP to about 50%LEP). This would suggest, that for this range at least, existing spending levels tend to be adequate and even where LEP funds are allocated they tend to be absorbed into the pool of general instructional expenditures and not reflected as differential costs of serving LEP children.

Most interesting, however, is the suggestion in Chart 1 that there comes a point (50 to 70%) where costs of serving additional LEP children are explosive. This pattern is somewhat contrary to our standard economic expectation of economies of scale by which marginal costs decrease with the scale of production. Yet, this outcome may be quite reasonable where students generally perceived to have a "minority" educational need are suddenly in a majority in a system for which the core technologies are not designed to serve that majority efficiently.

Table 9 and Chart 2 display the results of the sensitivity simulation for the percent of economically disadvantaged students in a district. As with LEP students Texas allocates additional linear weighted funds (20%/pupil identified) to aid in serving the special needs of these pupils. Again, the responsiveness of instructional expenditures was modeled. In this case, GMDH adheres to the linear responsiveness of cost to proportions of economically disadvantaged students. The question that remains is whether the magnitude of the slope of the relationship in the optimized GMDH model is comparable to that of the Texas allocations. Recall again that if the slope magnitude is less, it simply implies that some of the funds allocated for the purpose of serving these children are randomly dispersed and not showing up in the pattern of differentiated costs attributed to them. Conversely, if the magnitude of GMDH predicted differences were greater than the Texas allocation, the implication would be that districts are systematically providing supplements attributable to the needs of economically disadvantaged students (or at least associated with their existence) above and beyond state supplements.

Table 10 provides a more concrete comparison of the outcomes of the GMDH simulation with the current Texas compensatory weighting. In the GMDH simulation an additional 10% economically disadvantaged students yields an additional 1.19% average cost across all pupils. Applying this average marginal increase in cost to an average size district with varying proportions of economically disadvantaged students yields somewhat less compensatory funding per identified pupil than the current Texas allocation. The implication of this result is that the current Texas allocation for this particular subcategory of compensatory aid exceeds the cost, at least the predicted additional instructional cost per pupil, of serving these pupils.

4. Conclusions and Implications

In this study, GMDH did not display the same overwhelming predictive advantage that it had in earlier studies with different cross sectional (Baker, 1999) and time series (Baker, 2000) data sets. With non-sample predictive advantage as a preliminary measure of model fitness, this result does call into question the usefulness of the GMDH results in the various cost prediction simulations. It should be noted, however, that GMDH did produce modest gains in the prediction of total expenditures per pupil and produced comparable results to OLS in the prediction of instructional expenditures.

As for the actual results generated by the GMDH models in this study, a particularly important element is that the responsiveness of the output measure need not be uniform across the ranges of inputs and outputs or across cases in the data set. The combined ability of GMDH to identify non-linearities and complex interactions while producing accurate cost predictions may ultimately prove useful for generating more refined cost-differentiated funding formulas.

Ultimately we may be able to use methods such as GMDH to identify how a mix of particular school attributes affects costs, rather than simply assuming the total cost to be the sum of the effects of the individual attributes. The ability to make such judgements would eventually allow us to fund schools according to a predicted aggregate school need index and diminish our reliance on potentially inefficient pupil weighting schemes.

In this study, GMDH has also proven useful for revealing the variance in cost response at different levels of an exogenous input - proportion of limited English proficient students. Neural Networks are frequently touted as superior forecasting tools for their ability to identify turning points (Hansen and Nelson, 1997). In this case the turning point is not a change in the direction of a time series, but an apparent critical threshold level at which costs of serving LEP students may become explosive given the current teaching technologies of the system. The policy implications of such a finding might be either (1) that we must find a way to increase financial compensation at these critical levels or (2) we must seek to understand why these costs become explosive at these levels and restructure the system for greater efficiency. It should be noted that use of GMDH does not preclude the possibility of a uniform linear response where reasonable as seen in the results of the simulation involving the exogenous factor - proportion of economically disadvantaged students.

Finally, we emphasize that the objective of testing the methodologies herein is not to generate generalizable estimates of the relationship of performance to cost, or estimates of other exogenous factors to cost to be applied to other contexts. The reasoning behind flexible estimation differs quite significantly from traditional econometric modeling. Where the rationale behind construction of the traditional econometric model is most often to present and deductively test a theory-driven model of economic phenomena, the mindset driving flexible estimation and predictive modeling is more inductive and therefore context specific. The goal is to characterize, by a given set of information, the nature of the relationships within an observed system and to do so accurately and precisely. Thus, the findings of this study support the potential usefulness of the process of flexible modeling applied to cost prediction, not the resultant models found in the appendices. The process is generalizable, but the models are context specific.

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Table 1. Descriptive Statistics for Training and Production Set Data

	Training and Test Sets (N = 684)				**Production Set (N = 100)**			
	Mean	**Max**	**Min**	**CV**	**Mean**	**Max**	**Min**	**CV**
Enrollment	4,205	76,606	76	208	5,696	157,622	134	317
Percent of Students in High School	23	74	10	25	23	10	40	22
Percent Economically Disadvantaged	46	100	0	40	44	97	5	42
Percent Receiving Special Education Services	14	39	4	29	14	24	7	25
Percent Limited English Proficient	6	65	0	148	6	58	0	143
Percent Gifted and Talented	8	24	0	42	8	21	0	44
Teacher Cost Index	.86	1.03	.66	8.12	.86	1.03	.75	8.27
TAAS Composite Score 1998	82	98	44	11	83	97	48	11
Lagged TAAS Composite Score 1995	72	100	14	15	73	93	46	13
ACT Score	20	25	14	8.4	20	24	15	7.9
Per Pupil Expenditures (Total) 1998	6,085	24,145	4,118	24	5,861	10,700	4,297	20
Instructional Expenditures per Pupil 1998	3,178	8,782	2,061	21	3,195	5,104	2,229	18

Table 2. Linear Model Estimates

	Total Expenditures				Instructional Expenditures			
Variable	**OLS**		**2SLS**		**OLS**		**2SLS**	
	Estimate	*t-value*	Estimate	*t-value*	Estimate	*t-value*	Estimate	*t-value*
Intercept	9.72*	25.19	-0.98*	2.42	8.69*	26.67	1.10	0.58
Log Enrollment	-0.51*	-13.14	-0.35	0.98	-0.44*	-13.94	-0.35*	-5.30
Log of Enrollment Squared	0.03*	11.50	0.02*	3.08	0.02*	12.16	0.02*	4.53
Percent of Students in High School	0.36*	3.30	0.26	0.99	0.40*	4.17	0.35	1.83
Percent Economically Disadvantaged	0.06	1.17	0.31	1.78	0.14*	3.43	0.29*	2.39
Percent Receiving Special Education Services	0.50*	3.45	0.52	1.53	0.46*	3.72	0.48	1.93
Percent Limited English Proficient	0.24*	2.84	1.84*	3.75	0.02	0.33	1.20*	3.15
Percent Gifted and Talented	0.06	0.35	0.13	0.33	0.48*	3.32	0.38	1.32
Teacher Cost Index	0.18	1.68	0.05	0.19	0.07	0.78	0.02	0.09
Log of TAAS Composite Score 1998	0.09	1.21	4.44*	3.65	0.08	1.17	3.13*	3.34
Log of Lagged TAAS Composite Score 1995	0.01	0.16	-2.06*	-2.26	0.05	1.12	-1.34*	-2.02
Log of ACT Score 1998	0.12	1.44	-	-	0.13	1.92	-	-

*p<.05

Table 3. Measures of Fitness and Prediction Accuracy

	Predicting Total Expenditures		**Predicting Instructional Expenditures**	
	OLS	**GMDH**	**OLS**	**GMDH**
Training R-Squared	.49	.53	.55	.60
Production R-Squared	.33	.35	.45	.43
MAPE	10.2	9.1*	9.2	9.3
SD of MAPE	9.0	9.3	7.3	7.6

*Statistically significant difference ($p<.05$)

Table 4. GMDH Incremental Performance Increase Simulation (Total Expenditures)

	Mean	SD	Median	Max	Min	95%ile	5%ile
Actual	$ 5,862	$ 1,152	$ 5,573	$ 10,700	$ 4,297	$ 8,198	$ 4,622
Predicted	5,820	825	5,486	8,895	4,873	7,548	5,035
10%	5,816	818	5,486	8,896	4,865	7,512	5,036
20%	5,804	803	5,491	8,895	4,728	7,355	5,027
30%	5,785	800	5,471	8,907	4,156	7,254	5,018
40%	5,765	807	5,478	8,948	3,411	7,204	5,026
50%	5,752	823	5,525	8,974	2,643	7,177	5,034

Table 5. GMDH Performance Standards Simulation (Total Expenditures)

	Mean	SD	Median	Max	Min	95%ile	5%ile
Actual	$ 5,862	$ 1,152	$ 5,573	$ 10,700	$ 4,297	$ 8,198	$ 4,622
Predicted	5,940	945	5,629	9,858	4,457	7,734	5,112
50%ile	5,931	918	5,626	9,801	4,850	7,636	5,119
75%ile	5,955	921	5,640	9,834	4,936	7,661	5,133
95%ile	6,009	929	5,676	9,911	5,060	7,721	5,183

Table 6. GMDH Incremental Performance Increase Simulation (Instructional Expenditures)

	Mean	SD	Median	Max	Min	95%ile	5%ile
Actual	$ 3,195	$ 579	$ 3,138	$ 5,104	$ 2,229	$ 4,267	$ 2,484
Predicted	3,156	479	3,039	4,966	2,563	4,347	2,691
10%	3,143	495	3,015	4,913	2,286	4,381	2,705
20%	3,161	516	3,027	4,974	2,157	4,398	2,714
30%	3,172	531	3,038	5,051	2,087	4,393	2,708
40%	3,175	536	3,055	5,092	2,087	4,389	2,708
50%	3,177	537	3,066	5,092	2,087	4,400	2,660

Table 7. GMDH Performance Standards Simulation (Instructional Expenditures)

	Mean	SD	Median	Max	Min	95%ile	5%ile
Actual	$ 3,195	$ 579	$ 3,138	$ 5,104	$ 2,229	$ 4,267	$ 2,484
Predicted	3,156	479	3,039	4,966	2,563	4,347	2,691
50%ile	3,128	482	3,024	4,924	2,350	4,391	2,680
75%ile	3,135	497	3,024	4,966	2,253	4,388	2,689
95%ile	3,149	515	3,046	5,017	2,167	4,390	2,668

Table 8. Responsiveness of Cost to Changes in LEP (Holding TAAS98 at Median)

	Mean	SD	Median	Max	Min	95%ile	5%ile
Actual	$ 3,195	$ 579	$ 3,138	$ 5,104	$ 2,229	$ 4,267	$ 2,484
Predicted Base	3,146	537	3,019	6,302	2,452	4,059	2,683
0%LEP	3,090	517	2,977	6,252	2,509	3,977	2,661
10%LEP	3,153	528	3,037	6,378	2,560	4,057	2,715
20%LEP	3,124	523	3,009	6,319	2,537	4,020	2,690
30%LEP	3,054	511	2,942	6,179	2,480	3,930	2,630
40%LEP	2,995	501	2,885	6,059	2,432	3,854	2,579
50%LEP	2,993	501	2,883	6,056	2,431	3,852	2,577
60%LEP	3,099	519	2,985	6,269	2,516	3,988	2,668
70%LEP	3,377	565	3,253	6,832	2,742	4,346	2,908

Chart 1. Non-linear Responsiveness of Cost to LEP Changes

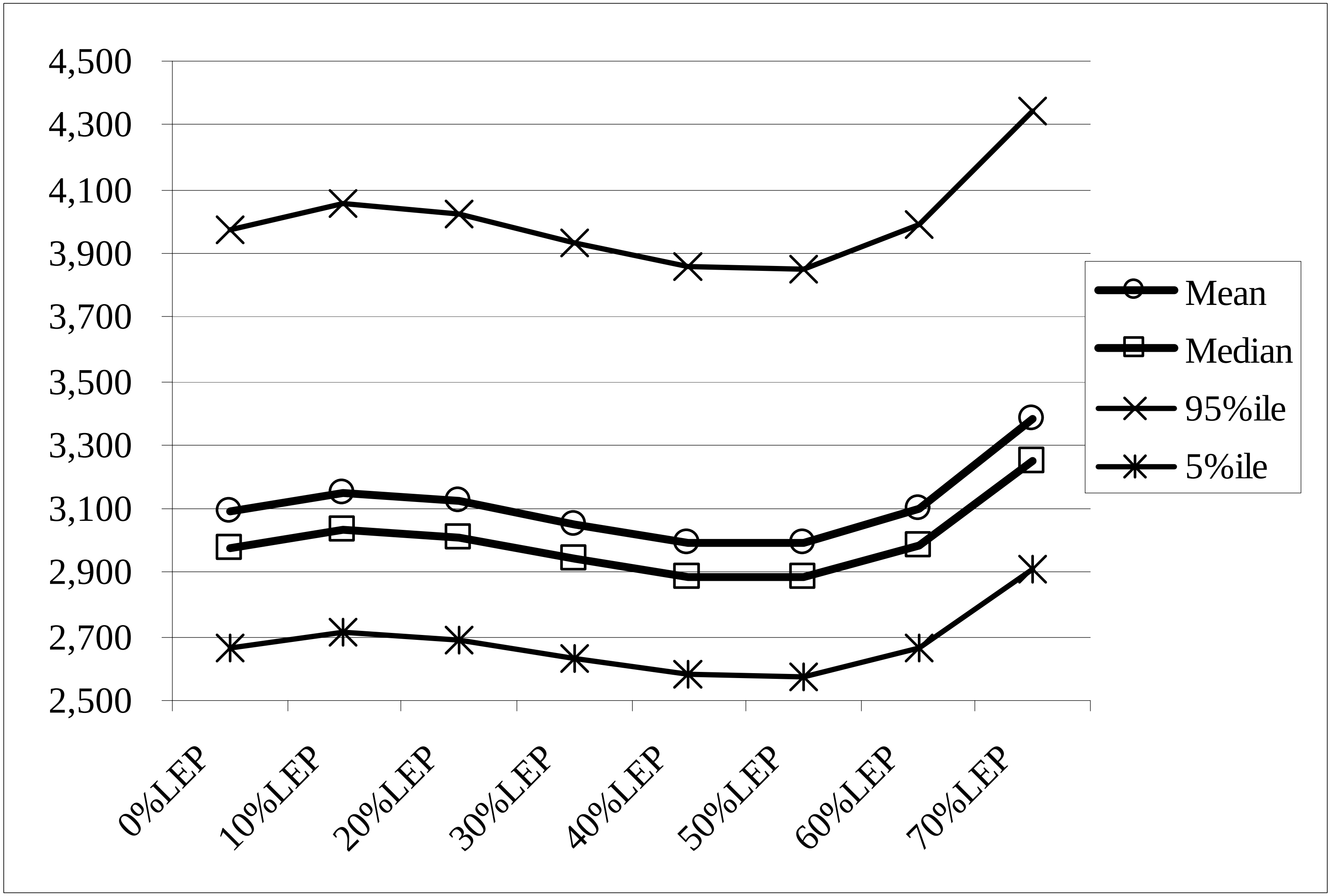

Table 9. Responsiveness of Cost to Different Levels of Economically Disadvantaged Students (Holding TAAS98 at Median)

	Mean	**SD**	**Median**	**Max**	**Min**	**95%ile**	**5%ile**
Actual	$ 3,195	$ 579	$ 3,138	$ 5,104	$ 2,229	$ 4,267	$ 2,484
Predicted Base	3,146	537	3,019	6,302	2,452	4,059	2,683
0%	2,958	479	2,863	5,803	2,492	3,833	2,571
10%	2,993	485	2,897	5,872	2,522	3,878	2,601
20%	3,029	491	2,932	5,942	2,552	3,925	2,632
30%	3,065	497	2,967	6,013	2,582	3,972	2,664
40%	3,102	502	3,002	6,085	2,613	4,019	2,695
50%	3,139	508	3,038	6,157	2,644	4,067	2,728
60%	3,176	515	3,074	6,231	2,676	4,115	2,760
70%	3,214	521	3,111	6,305	2,708	4,165	2,793
80%	3,253	527	3,148	6,380	2,740	4,214	2,826
90%	3,291	533	3,186	6,457	2,773	4,264	2,860
100%	3,331	540	3,224	6,534	2,806	4,315	2,894

Chart 2. Responsiveness of Cost to Different Levels of Economically Disadvantaged Students (Holding TAAS98 at Median)

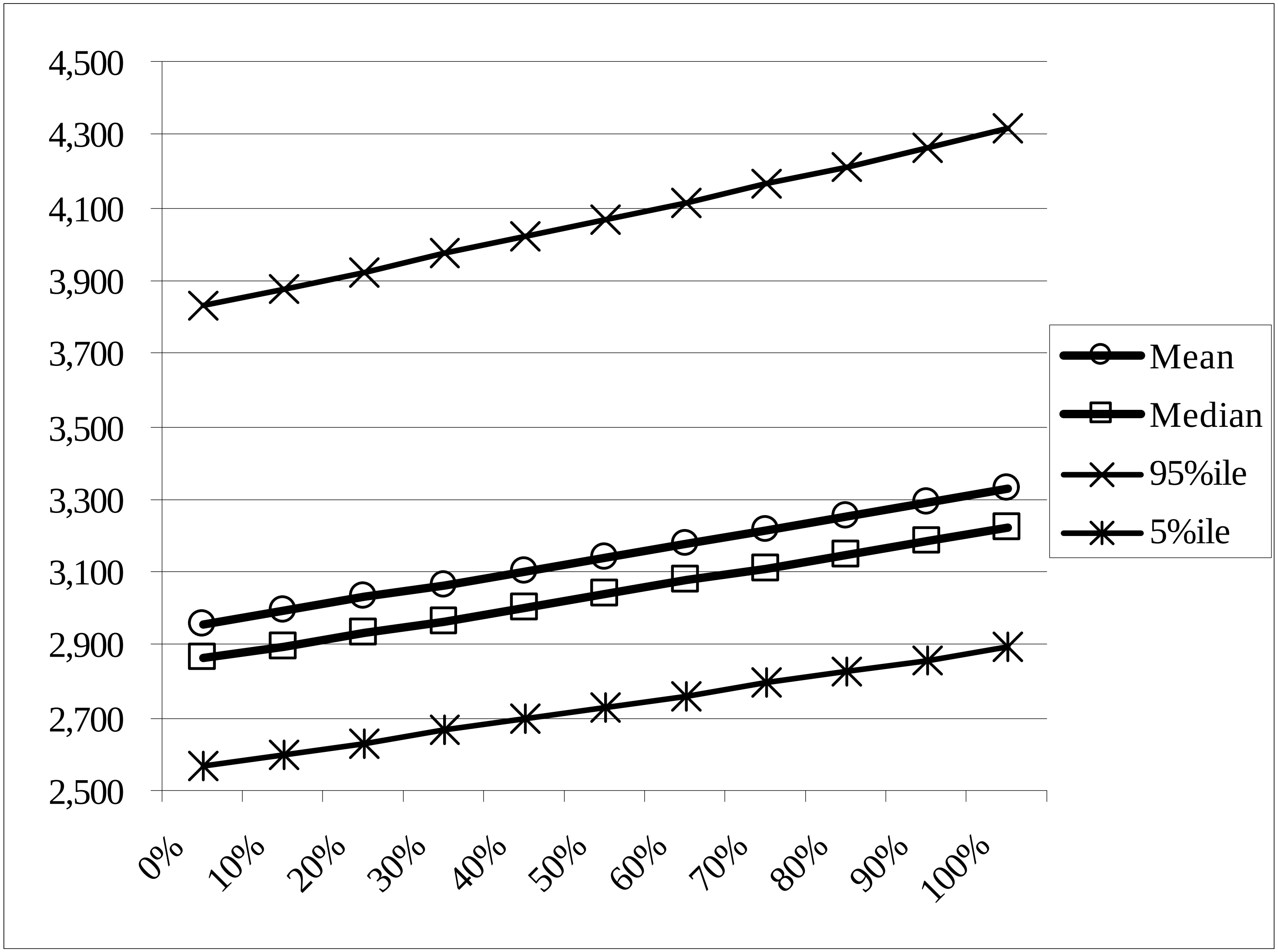

Table 10. Comparison of GMDH Predictions and Texas Compensatory Weighting

GMDH Predicted System	**10% Comp.**	**20% Comp.**	**30% Comp.**	**40% Comp.**	**50% Comp.**
District Enrollment	5,000	5,000	5,000	5,000	5,000
Foundation (1998 - 99)	2,396	2,396	2,396	2,396	2,396
Number Economically Disadvantaged	500	1,000	1,500	2,000	2,500
1.19% Foundation(a)	2,425	2,454	2,483	2,512	2,542
Adjusted Funding	12,122,921	12,267,548	12,413,900	12,561,998	12,711,862
Base Funding	11,980,000	11,980,000	11,980,000	11,980,000	11,980,000
Compensatory Funding	142,921	287,548	433,900	581,998	731,862
Compensatory Funding per Compensatory Pupil	*286*	*288*	*289*	*291*	*293*

Texas Compensatory Aid	**10% Comp.**	**20% Comp.**	**30% Comp.**	**40% Comp.**	**50% Comp.**
District Enrollment	5,000	5,000	5,000	5,000	5,000
Foundation (1998 - 99)	2,396	2,396	2,396	2,396	2,396
Number Economically Disadvantaged	500	1,000	1,500	2,000	2,500
ADA Adjustment (.2 * Number Disadvantaged)	100	200	300	400	500
Compensatory Funding	239,600	479,200	718,800	958,400	1,198,000
Base Funding	11,980,000	11,980,000	11,980,000	11,980,000	11,980,000
Adjusted Funding	12,219,600	12,459,200	12,698,800	12,938,400	13,178,000
Compensatory Funding per Compensatory Pupil	*479*	*479*	*479*	*479*	*479*

APPENDIX A
GMDH MODEL EQUATIONS

Total Expenditures Model

Best formula:

Y=3.6E-002*X10-5.6E-002*X3+6.6E-002*X8-5.9E-002*X9+0.24*X5-0.58*X1+0.1*X4-0.19+7.2E-002*X2+7.5E-002*X7+0.45*X1^2+0.32*X7^2+8.5E-002*X9^2-8.E-002*X1^3-0.26*X7^3+6.3E-002*X9^3-0.16*X1*X9+0.11*X7*X9+0.23*X1*X7*X9+7.4E-002*X2^2+3.4E-002*X4^2-1.9E-002*X2^3+6.3E-002*X2*X4+0.12*X3^2+0.11*X3^3-0.25*X3*X5-0.13*X8^2-8.E-002*X10^2+0.25*X8*X10

Legend: X1=2.*(ENR98-4.75)/5.2-1.
X2=2.*(PHS-.11)/.22-1.
X3=2.*(FRLUN98-.09)/.75-1.
X4=2.*(SPEC98-.06)/.16-1.
X5=2.*(LEP98+.12)/.36-1.
X6=2.*(GT98-.01)/.13-1.
X7=2.*(TCI94-.72)/.28-1.
X8=2.*(TAS98-4.15)/.49-1.
X9=2.*(TAS95-3.94)/.64-1.
X10=2.*(ACT98-2.81)/.34-1.
Y=2.*(PPE98-8.33)/.74-1.

Instructional Expenditures Model

Best formula:

Y=-0.17+3.3E-002*X4+7.9E-002*X5-5.9E-002*X7+0.13*X3+7.3E-002*X8+8.1E-002*X6-0.5*X1+9.6E-002*X2+0.42*X1^2+8.4E-002*X2^2+8.9E-003*X4^2-8.7E-002*X1^3-1.9E-002*X6^3-7.9E-002*X2*X6-0.27*X1*X2*X6+5.1E-002*X2^2*X6+3.2E-002*X2*X4*X6+0.24*X1^2*X2*X6+4.4E-002*X2^3*X6+4.5E-002*X2*X4^2*X6-4.6E-002*X1^3*X2*X6+0.17*X8^2+0.11*X8^3+0.11*X7^2+4.7E-002*X4^3-7.5E-002*X4*X7-0.15*X1*X9-0.16*X5^2+4.4E-002*X5^3+8.5E-002*X7*X9-6.9E-002*X9^2+7.8E-002*X8*X9

Legend: X1=2.*(enrol-4.71)/5.31-1.
X2=2.*(hsp98-.11)/.23-1.
X3=2.*(frlun98-.09)/.75-1.
X4=2.*(spec98-.06)/.16-1.
X5=2.*(lep98+.12)/.36-1.
X6=2.*(gt98-.01)/.13-1.
X7=2.*(tci94-.72)/.28-1.
X8=2.*(tas98-4.16)/.48-1.
X9=2.*(tas95-3.93)/.65-1.
X10=2.*(act98-2.81)/.34-1.
Y=2.*(inpp98-7.7)/.7-1.