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USING GMDH FOR MODELING ECONOMICAL INDICES OF MINE OPENING

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Dependency of explicit costs index of mining opening on its parameters is modeled. In addition, the problem of structural identification is solved. Models enumeration is realized with the help of group methods of data handling (GMDH). Least-squares and least-modules methods are used for evaluating model parameters. The quality of resulting models is evaluated by the following criteria: (a) remaining sum of squares criteria; (b) "sliding control" criteria; (c) regularity criteria.

Keywords: Modeling; GMDH-technique; Iterative algorithm; Economic parameters; Nonlinear regress

1. INTRODUCTION

A static object is considered (mountain range, undercut by excavation) with *m* entries (rocks characteristics, fissuring attributes, depth of opening, etc.) and one output (mining opening maintenance explicit costs index). Object examination results are represented as matrix X [*N* x *m*] and vector y [*N* x 1]. The problem of structural identification [1,2] has to be solved, using the data of *N* examinations, i.e. the structure of one-dimensional variable *y* dependency on the collection of entry variables. X has to be determined under conditions that it is not known *a priori* which factors (parameters, affecting the stability of a mining opening) exactly, from the collection of entry variables, take part in forming the output variable y.

Let the desired object model belong to a set G, containing models of the following look:

$$\hat{\mathbf{y}} = f(X, \hat{Q}_f), \tag{1}$$

where $\hat{Q}_f$ is a vector of model parameters, evaluated somehow by the examination data. At that, the problem of structural identification is reduced to determining the

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minimum of specified model quality criterion J:

$$f^* = \arg\min_{f \in \mathbf{G}} J(\mathbf{y}, f(X, \hat{Q}_f)) \tag{2}$$

Methods for solving the problem may differ at least in the following features: (a) algorithm of models structures forming (generation) from the set G; (b) these models' parameters evaluation methods; (c) quality evaluation criterion J itself; (d) organizing this criterion's movement to minimum. Let us note that model parameter evaluation methods, their quality criteria and methods for searching criteria's minimums are independent in general and can be applied in different combinations. That is why, a lot of methods for solving the mentioned problem can be suggested (2). For building up a mathematical model of explicit costs index of mining opening dependency on its parameters we will use:

(a) GMDH as a method for enumerating models;
(b) Least-square and least-module methods for evaluating model parameters;
(c) Remaining sum of squares criteria, "sliding control" criteria and regularity criteria for evaluating quality of resulting models.

Model structure is chosen in accordance with principles of economy and adequacy.

2. MATHEMATICAL PROBLEM STATEMENT

Let the functioning principle of the examined object have the following look:

$$\mathbf{y} = \overset{o}{y} + \xi = \sum_{j=1}^{\overset{o}{m}} \overset{o}{\mathbf{\Theta}}_j \overset{o}{x}_j + \xi, \tag{3}$$

where y is the object's observed output; $\overset{o}{y}$ is the object's unobserved unnoisy output; ξ is an unobserved accidental metering mistake; $\overset{o}{x}_j$ is the jth object's entry from the set of entries $\overset{o}{\chi}$, taking part in forming the object's output; $\overset{o}{m}$ is the number of entries, belonging to the set $\overset{o}{\chi}$; $\mathbf{\Theta} = (\overset{o}{\Theta}_1, \overset{o}{\Theta}_2, \ldots, \overset{o}{\Theta}_{\overset{o}{m}})$ is a vector of nonzero unknown coefficients.

The set of entries $\overset{o}{\chi}$ is unknown. We only know that $\overset{o}{\chi} \subset \chi$, where χ is some set of exactly measured m entries of the object.

Let as a result of examination we have:

1. X - matrix (of dimension N x m) of N examinations of m entries of the set χ, range $X = m$,
2. y vector (of dimension N x 1) of corresponding examinations of the output variable y.

According to the object's functioning principle (3) the following equality takes place:

$$\mathbf{y} = \overset{o}{\mathbf{y}} + \xi = \overset{o}{\mathbf{X}} \cdot \overset{o}{\mathbf{\Theta}} + \xi, \tag{4}$$

where $\overset{o}{\mathbf{y}}$ is a vector (of dimension $N \times 1$) of object's unobserved unnoisy output values, $\overset{o}{\mathbf{X}}$ is a matrix (of dimension $N \times \overset{o}{m}$) of examination of object's values, belonging to the set $\overset{o}{\chi}$; ξ is a vector (of dimension $N \times 1$) of unobserved accidental metering mistakes.

Let the following assumptions be true relative to ξ

$$E(\xi) = \boldsymbol{O}_N, \qquad E\{\xi\xi^T\} = \sigma^2 \mathbf{I}_N,$$

where E is a statistical expectation sign of; $\boldsymbol{O}_N$ is a zero vector (of dimension $N \times 1$); T is a transposition sign; σ^2 is an unknown quantity; $\mathbf{I}_N$ is a unitary matrix (of dimension $N \times N$).

Now we have to find: (1) $\overset{o}{\chi}$ set; (2) coefficients' vector evaluation $\overset{o}{\boldsymbol{\Theta}}$; (3) telemetry errors dispersion evaluation σ^2.

3. STRUCTURAL IDENTIFICATION ALGORITHM

Synthesized models class has the following look:

$$\hat{\mathbf{y}} = \sum_{q=1}^{s} \Theta_q \prod_{j=1}^{m} x_j^{\alpha_{qj}}, \tag{5}$$

where $\hat{\mathbf{y}}$ is an output variable; s is the number of model members; Θ_q, $q = 1, 2, \ldots, s$ are coefficients, x_j, $j = 1, 2, \ldots, m$ are entry variables; m is the number of entry variables; α_{q_j} is an exponent of power in which the Xj variable is contained in the qth member.

Definition 1 Quotient description is a vector $\hat{\mathbf{z}}$ (of dimension $N \times 1$), obtained at some iteration of the algorithm as vector y approximation.

Definition 2 Quotient description structure is a collection of parameters α_{q_j} and s, defining $\hat{\mathbf{z}}$ in representation (5).

For building up a GMDH algorithm it is necessary to:

1. select initial quotient descriptions matrix $\hat{\mathbf{Z}}^o$;
2. define operator R, realizing mapping

$$\hat{\mathbf{Z}}^{r-1} \xrightarrow{R} \hat{\mathbf{Z}}^{r},$$

   $r = 1, 2, \ldots$ - iteration number;
3. select a rule for finishing iterative process.

We will define the general form of quotient descriptions matrix $\hat{\mathbf{Z}}^r$ the following way:

$$\hat{\mathbf{Z}}^r = [\hat{\mathbf{z}}_1^r \vdots \hat{\mathbf{z}}_2^r \vdots \ldots \vdots \hat{\mathbf{z}}_{F+2+m+2s}^r], \tag{6}$$

where $\hat{\mathbf{z}}_j^r$, $j = 1, 2, \ldots, F + 2 + m + 2s$ are vectors (of dimension $N \times 1$), quotient descriptions; F is the number of the best quotient descriptions that are passed from iteration to iteration; s is the number of members in the structure of the $(r-1)$th iteration's best quotient description.

Let us denote

$$\hat{\mathbf{G}}^r = [\hat{\mathbf{z}}_1^r \vdots \hat{\mathbf{z}}_2^r \vdots \ldots \vdots \hat{\mathbf{z}}_F^r],$$
$$\hat{\mathbf{C}}^r = [\hat{\mathbf{z}}_{F+1}^r \vdots \hat{\mathbf{z}}_{F+2}^r \vdots \ldots \vdots \hat{\mathbf{z}}_{F+2+m}^r], \tag{7}$$
$$\hat{\mathbf{D}}^r = [\hat{\mathbf{z}}_{F+2+m+1}^r \vdots \hat{\mathbf{z}}_{F+2+m+2}^r \vdots \ldots \vdots \hat{\mathbf{z}}_{F+2+m+2s}^r]$$

The algorithm consists of the following steps:

1. Initial quotient descriptions matrix $\hat{\mathbf{Z}}^o$ is selected (it is supposed that, $s = 0$ for this matrix):

$$\hat{\mathbf{Z}}^{\mathbf{o}} = [\mathbf{O} \vdots \boldsymbol{o} \vdots \boldsymbol{I} \vdots \mathbf{X}] = [\mathbf{O} \vdots \hat{\mathbf{C}}^{\mathbf{o}}], \tag{8}$$

where $\mathbf{O}$ is a zero matrix (of dimension N x '); $\boldsymbol{o}$ is a zero vector (of dimension N x 1); $\boldsymbol{I}$ is a unit vector (of dimension N x 1); $\mathbf{X} = [x_1 \vdots x_2 \vdots \ldots \vdots x_m]$ is a matrix of entry parameters examinations (of dimension N x m).

2. Operator R is defined.

Let vectors $\hat{\mathbf{z}}^r$ be defined by the following rule:

$$\hat{\mathbf{z}}^r(i) = \hat{a}\hat{\mathbf{z}}_{j_1}^{r-1}(i) + \hat{b}\hat{\mathbf{z}}_{j_2}^{r-1}(i)\hat{\mathbf{z}}_{j_3}^{r-1}(i), \tag{9}$$

where $r = 1, 2, \ldots$ is the number of iteration; $i = 1, 2, \ldots, N$ is the number of examination; $j_1, j_2, j_3 = 1, 2, \ldots, F + 2 + m + s$, $(j_3 \geq j_2)$ are the numbers of quotient descriptions from the matrix $\hat{\mathbf{Z}}^{r-1}$; $\hat{a}$, $\hat{b}$ are coefficients, defined on the learning subsample of examinations (A).

Values of coefficients $\hat{a}$, $\hat{b}$ are determined as a solution of the minimization problem:

$$\hat{a}, \hat{b} = \arg\min_{a,b} \Phi(a, b),$$
$$\Phi(a, b) = \sum_{i=1}^{N(A)} e_A^2(i), \tag{10}$$

where $e_A(i)$, $i = 1, 2, \ldots, N(A)$, are remainders in regression y_A by two variables:

$$y_A(i) = a\hat{\mathbf{z}}_{j_1 A}^{r-1}(i) + b\hat{\mathbf{z}}_{j_2 A}^{r-1}(i)\hat{\mathbf{z}}_{j_3 A}^{r-1}(i) + e_A(i) \tag{11}$$

From all the quotient descriptions, obtained by Eqs. (9)-(1 1) we will select F descriptions, that are the best by the minimum of remainders quadratic form on the checking subsample of examinations *(B)*:

$$J = \frac{1}{N(B)} \sum_{i=1}^{N(B)} \hat{e}_B^2(i),$$
$$\hat{e}_B(i) = y_B(i) - \hat{\mathbf{z}}_B^r(i), \tag{12}$$
$$\hat{\mathbf{z}}_B^r(i) = \hat{a}\hat{\mathbf{z}}_{j_1 B}^{r-1}(i) + \hat{b}\hat{\mathbf{z}}_{j_2 B}^{r-1}(i)\hat{\mathbf{z}}_{j_3 B}^{r-1}(i),$$

where $N(B)$ is a volume of the checking subsample B.

Ranged by decreasing of J quantity, selected best quotient descriptions are used when forming the matrix

$$\hat{\mathbf{G}}^r = [\hat{\mathbf{z}}_1^r \vdots \hat{\mathbf{z}}_2^r \vdots \ldots \vdots \hat{\mathbf{z}}_F^r]. \tag{13}$$

Matrix $\hat{\mathbf{C}}^r$ is not changed: $\hat{\mathbf{C}}^r = \hat{\mathbf{C}}^{r-1}$. Matrix $\hat{\mathbf{D}}^r$ is formed taking into account the structure of the best of F selected quotient descriptions ($\hat{\mathbf{z}}_F^r$). First s columns of matrix $\hat{\mathbf{D}}^r$ are filled with separate members of best quotient description by the following rule:

$$d_h^r(i) = \hat{\mathbf{z}}_{F+2+m+h}^r(i) = \Theta_h \prod_{7=1}^{m} x_j^{\alpha_{h_j}}(i), \tag{14}$$

where $h = 1, 2, \ldots, s$ is the number of member in the structure; s is the number of members in the structure of best quotient description.

Second s columns of matrix $\hat{\mathbf{D}}^r$ are formed by byturn exclusion of separate members from the structure of best quotient description by the following rule:

$$\hat{d}_{s+h}^r(i) = \hat{\mathbf{z}}_{F+2+m+s+h}^r(i) = \sum_{\substack{q=1 \\ (q \neq h)}}^{s} \hat{\Theta}_q \prod_{j=1}^{m} x_j^{\alpha_{q_j}}(i) \tag{15}$$

Thus, operator R of mapping $\hat{\mathbf{Z}}^{r-1} \xrightarrow{R} \hat{\mathbf{Z}}^r$ is defined.

3. The rule for finishing iterative scheme: calculations are finished at the r^*th iteration if

$$J(\hat{\mathbf{z}}_F^{r^*-1}) - J(\hat{\mathbf{z}}_F^{r^*}) < \delta_r, \tag{16}$$

where $J(\hat{\mathbf{z}}_F^{r^*})$ is the value of rth iteration's best quotient description criterion; δ_r is a given number.

The feature of the algorithm is its multistageness. The number of current stage defines the maximal allowed number of members in models. Models synthesis starts at the stage with number p — \ or from any specified number p^o. Each stage is an iterative scheme (8) (16). The finite matrix of quotient descriptions from the previous stage determines initial matrix of quotient description of the stage with number p.

$$\hat{\mathbf{Z}}_{p}^{o} = \hat{\mathbf{Z}}_{p-1}^{r^*}, \tag{17}$$

and for $p = p^o$ it coincides with (8). Calculations finish at the stage p^* if

$$J(\hat{\mathbf{z}}_{F,p^*-1}^{r^*}) - J(\hat{\mathbf{z}}_{F,p^*}^{r^*}) < \delta_p, \tag{18}$$

where $-J(\hat{\mathbf{z}}_{F,p}^{r})$ is the value of the pth stage's rth iteration's best quotient description criteria; δ_p is a specified number.

Distinctive features of the algorithm are: (1) multistage search of a model; (2) the model is searched in both linearly and nonlinearly by entry variable model classes; (3) technique of excluding separate members of best quotient description and resulting broadening the basis collection of arguments; (4) GMDH scheme of calculating the moving examination criterion, optimal of computational expenses for iterative algorithms; (5) possibility to evaluate coefficients in models by both least square and least module methods.

4. RESULTS OF MODELING MINE OPENINGS ECONOMICAL INDICES WITH STRUCTURAL IDENTIFICATION ALGORITHM

Examinations data are taken from the reports about excavations of mines "Belozerskaya" and "RKKA" of industrial complex "Dobropolyeugol" (Ukraine): 25 examinations of the mine "Belozerskaya" and 34 examinations of the mine "RKKA".

Each opening is characterized by 14 parameters:

1. costs (in hrivnyas) of 1 m of mine roadway;
2. depth of location (m);
3. length (km);
4. azimuth of direction (degrees);
5. average water entry (m/h);
6. step of shoring (m);
7. compressive resistance of main roof rocks (Pa);
8. compressive resistance of immediate roof rocks (Pa);
9. compressive resistance of immediate soil rocks (Pa);
10. compressive resistance of main soil rocks (Pa);
11. rock cracks dip azimuth (degrees);
12. rock cracks dip angle (degrees);
13. coal cracks dip azimuth (degrees);
14. coal cracks dip angle (degrees).

Dependency of explicit costs index of mining opening on its parameters 2-14 is modeled.

The scheme of carrying the computations is conditioned on the features of considered problem:

- absence of *a priori* information about model structure and dispersion of observational errors (which justifies the usage of GMDH algorithms);
- large number ($m = 13$) of entry variables (parameters 2-14) when the number ($N = 25 + 34 = 59$) observations is low (which allows preferring GMDH iterative algorithms to algorithms of combinatorial kind).

Computations were carried out for three samples:

1. examinations data for shaft "Belozerskaya" (marked by *"B")*;
2. examinations data for shaft "RKKA" (marked by "*R*");
3. examinations data for both shafts (marked by *"B+R")*.

In all three cases two kinds of model structure were calculated: linear (by entry variables) and nonlinear. For building up nonlinear models the data were centered and normalized.

During calculations for resulting models quality evaluation remaining sum of squares criteria was applied first:

$$J(s) = ||y - f(X, \hat{\Theta}(s))||^2,$$

where s is a model complexity, i.e. the number of evaluated parameters.

It allowed to approximately evaluate model complexity, because $J(s)$ decreases with increasing model complexity (i.e. the greater the number of model members, the less the $J(s)$). When the $J(s)$ deceleration rate (when the model complexity is increasing) sharply slows down, the number of model members can be considered to be sufficient. The next (after $J(s)$) was applied the $U(s)$ criterion ("sliding examine" criterion [2]), which may not decrease with increase in model complexity increasing but may have minimum, which corresponds to the model of best complexity.

Computations were carried out for three data samples *(B, R, B+R),* for model structure types (linear, nonlinear), for two model quality evaluation criteria (squares remaining sum, "sliding examine").

Resulting model quality was evaluated by traditional characteristics; mean square deviation of model values $\hat{y}$ from observed, as well as unambiguously bound with multiple correlation coefficient $R_{y,X}$.

In all resulting models the following notation was taken for sample parameters: y – cost of 1 m of mine roadway, x_1- depth of location, x_2 - length, x_3 azimuth of direction, x_4 - average water entry, x_5 - step of shorting, x_6 compressive resistance of main roof rocks, x_7 - compressive resistance of immediate roof rocks, x_8 - compressive resistance of immediate soil rocks, x_9 - compressive resistance of main soil rocks, x_{10} - rock cracks dip azimuth, x_{11} - rock cracks dip angle, x_{12} - coal cracks dip azimuth, x_{13} - coal cracks dip angle.

For models with $R_{y,X} > 0.6$, x_i parameters, included into the corresponding regression equations are enumerated in Table I.

The table shows which attributes can be seen more frequently in better models, qualitative figure of dependency $y = f(x_1, x_2, \ldots, x_{13})$ structure can be seen. For example, x_3 and x_{11} attributes (azimuth of direction and rock cracks dip angle)

TABLE I Results of modelling

Model structure type	*Examinations data*		
	"B"	*"RKKA"*	*"B+RKKA"*
Linear	$J(3)=0.62$; $R_{y,X}=0.8$	$J(4)=0.80$; $R_{y,X}=0.64$	$R_{y,X}<0.6$
	x_1, x_3, x_7	x_2, x_3, x_{10}, x_{11}	
	$U(4)=0.82$; $R_{y,X}=0.63$	$R_{y,X}<0.6$	$R_{y,X}<0.6$
	x_3, x_5, x_{11}, x_{12}	–	–
Nonlinear	$J(5)=0.63$; $R_{y,X}=0.78$	$J(4)=0.62$; $R_{y,X}=0.79$	$J(8)=0.54$; $R_{y,X}=0.85$
	x_3, x_5, x_8, x_{11}, x_{12}	x_3, x_8, x_{10}, x_{11}	x_1, x_2, x_3, x_5, x_6, x_8, x_9, x_{11}
	$U(3)=0.78$; $R_{y,X}=0.62$	$U(4)=0.79$; $R_{y,X}=0.71$	$U(6)=0.75$; $R_{y,X}=0.77$
	x_9, x_{11}, x_{12}	x_3, x_6, x_{10}, x_{12}	x_1, x_3, x_5, x_8, x_{11}, x_{13}

appear more frequently in models, than other attributes, moreover they are included in regression equations with relatively great coefficients. Hence, we can state for sure that attributes x_3 and x_{11} are the factors for modeled variable y. x_5 and x_8 attributes are of second importance (step of shorting and compressive resistance of immediate soil rocks).

If we want to choose a trade-off decision between quality and difficulty of models, the following models should be considered the best:

$$y = -4.14\,x_7 + 5.882\,x_3 + 3.095\,x_1,$$
$$y = -0.61\,x_3 x_8 x_{11}^2 + 0.73\,x_{10} x_{11} + 0.59\,x_{11} - 0.2,$$
$$y = -0.09\,x_{13} - 0.72\,x_{11} - 0.34\,x_5 + 0.87\,x_3.$$

For receiving good prognostic properties, obtained model has to be checked on examinations data samples at least for two mines. However, the qualitative structure of opening maintenance costs dependency on the enumerated parameters-attributes has been found. It is determined by the dependency

$$y = f(x_3, x_5, x_8, x_{11}).$$

5. CONCLUSIONS

Modeling results allowed to determined the structure of mine opening costs index dependency on the opening parameters. Two parameters exert the most influence on the costs index: azimuth of direction and rock cracks dip angle. The next most influential parameters are step of shoring and compressive resistance of immediate soil rocks. Investigations scientifically justify the fact that openings, located in the same lithologic sections, but drifted in opposite directions using the same drifting technology, have essentially different stability and require different economical costs. It is expedient to take into account the obtained results when designing mines.

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