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# Logistic GMDH-type Neural Networks and their Application to the Identification of the X-ray Film Characteristic Curve

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## ABSTRACT

In this paper, logistic Group Method of Data Handing (GMDH)-type neural networks identifying a complex nonlinear system are proposed. The logistic GMDH-type neural networks are automatically organized by using the heuristic self-organization method [1] which is used in GMDH method [2]-[6]. In the logistic GMDH-type neural networks, the structural parameters such as the number of layers, the number of neurons in each layer, useful input variables and optimum neuron architectures are automatically determined by using the error criterion derived from the AIC (Akaike's Information Criterion) [7]. This way, optimum neural network architectures which fit the complexity of the nonlinear system are produced. The logistic GMDH-type neural networks have been applied to the identification problem of the X-ray film characteristic curve. It has been found that the modeling with the logistic GMDH-type neural networks is more accurate than when multiple regression analysis, the conventional neural networks [8]-[10] and the GMDH method [6] are used.

## 1. INTRODUCTION

The GMDH-type neural networks have first been proposed [11]-[13]. They have several advantages compared with conventional neural networks [8]-[10]. They can automatically organize multilayered neural networks by using the heuristic self-organization method [1]. In the GMDH-type neural networks, many types of neurons, which are polynomial type, sigmoid function type, radial basis function type and like, can be used to organize neural network architectures and optimum neuron architectures are selected so as to fit the complexity of the nonlinear system by using the AIC [7].

In this study, the logistic GMDH-type neural networks are proposed. They are developed based on the GMDH-type neural network algorithm. In the logistic GMDH-type neural networks, different nonlinear combinations of input variables are generated and their useful combinations are selected so as to minimize the AIC. Therefore, the high order effects of the input variables are automatically included. The logistic GMDH-type neural networks have been applied to the identification problem of the X-ray characteristic curve and compared with the other models.

## 2. GMDH-TYPE NEURAL NETWORK ALGORITHM

The GMDH-type neural network algorithm can automatically develop the optimum neural network architectures by using the heuristic self-organization method [1]. The heuristic self-organization method in the GMDH-type neural networks is implemented through the following five stages:

**1) Separating the original data into training and test sets.**

The original data are separated into training and test sets. The training data are used for the estimation of the weights of the neural networks. The test data are used for organizing neural network architectures.

**2) Generating the combinations of the input variables in each layer.**

Many combinations of r input variables are generated in each layer. The number of combinations is p!/((p-r)!r!). Here, p is the number of input variables and the number of r is usually set to two.

**3) Selecting the optimum neuron architectures**

For each combination, the optimum neuron architectures which describe the partial characteristics of the nonlinear system can be calculated by applying the regression analysis [14] to the training data. The output variables ($y_k$) of the optimum neurons are called intermediate variables. In the GMDH-type neural networks, the optimum neurons are selected from different neuron architectures.

**4) Selecting the intermediate variables.**

The L intermediate variables which give the L smallest test errors which are calculated by using the test data are selected from the generated intermediate variables ($y_k$).

**5) Stopping the multilayered iterative computation.**

When the errors for the test data in each layer stop decreasing, the iterative computation is terminated. The complete neural networks which describe the characteristics of the nonlinear system can be constructed by using the optimum neurons which are generated in each layer.

In the GMDH-type neural networks, different neuron architectures can be used to organize neural networks which fit the complexity of the nonlinear system. The architectures used in the GMDH-type neural networks are shown in Fig.1. They are described by the following equations:

**1) First type neuron**

$\Sigma$: (Nonlinear function)

$$z_k = w_1u_i+w_2u_j+w_3u_iu_j+w_4u_i^2+w_5u_j^2+w_6u_i^3+w_7u_i^2u_j+w_8u_iu_j^2+w_9u_j^3 - w_0\theta_1 \quad (1)$$

f : (Nonlinear function)

$$y_k = 1/(1+exp(-z_k)) \quad (2)$$

**2) Second type neuron**

$\Sigma$: (Nonlinear function)

$$z_k = w_1u_i+w_2u_j+w_3u_iu_j+w_4u_i^2+w_5u_j^2+w_6u_i^3+w_7u_i^2u_j+w_8u_iu_j^2+w_9u_j^3 - w_0\theta_1 \quad (3)$$

f : (Linear function)

$$y_k = z_k \quad (4)$$

**3) Third type neuron**

$\Sigma$: (Linear function)

$$z_k = w_1u_1+w_2u_2+w_3u_3+\cdots+w_ru_r - w_0\theta_1 \quad (r<p) \quad (5)$$

f : (Nonlinear function)

$$y_k = 1/(1+exp(-z_k)) \quad (6)$$

**4) Fourth type neuron**

$\Sigma$: (Linear function)

$$z_k = w_1u_1+w_2u_2+w_3u_3+\cdots+w_ru_r - w_0\theta_1 \quad (r<p) \quad (7)$$

f : (Linear function)

$$y_k = z_k \quad (8)$$

**5) Fifth type neuron**

$\Sigma$: (Nonlinear function)

$$z_k = w_1u_i+w_2u_j+w_3u_iu_j+w_4u_i^2+w_5u_j^2+w_6u_i^3+w_7u_i^2u_j+w_8u_iu_j^2+w_9u_j^3 - w_0\theta_1 \quad (9)$$

f : (Nonlinear function)

$$y_k = exp(-z_k^2) \quad (10)$$

**6) Sixth type neuron**

$\Sigma$: (Linear function)

$$z_k = w_1u_1+w_2u_2+w_3u_3+\cdots+w_ru_r - w_0\theta_1 \quad (r<p) \quad (11)$$

f : (Nonlinear function)

$$y_k = exp(-z_k^2) \quad (12)$$

**7) Seventh type neuron**

$\Sigma$: (Linear function)

$$z_k = w_1u_1+w_2u_2+w_3u_3+\cdots+w_ru_r - w_0\theta_1 \quad (r<p) \quad (13)$$

f : (Nonlinear function)

$$y_k = a_0+a_1z_k+a_2z_k^2+\cdots+a_mz_k^m \quad (14)$$

Here, $\theta_1$=1 and $w_i$ (i=0,1,2,$\cdots$ ) are weights between the neurons. The optimum neuron architectures which fit the nonlinear system are automatically selected by using the AIC [7]. Therefore, many kinds of nonlinear systems can be automatically modeled by using the GMDH-type neural networks. Logistic GMDH-type neural networks can be developed based on above mentioned GMDH-type algorithm.

## 3. LOGISTIC GMDH-TYPE NEURAL NETWORKS

Logistic GMDH-type neural networks identifying a complex nonlinear system are proposed. The conventional logistic regression analysis can identify a nonlinear system. Fig.2 shows the architecture of the conventional logistic regression analysis. In a complex system, however, obtaining accurate identification by using conventional logistic regression analysis is difficult due to the absence of high order effect of the input variables. In the logistic GMDH-type neural networks, many kinds of nonlinear combinations of the input variables are generated and only useful nonlinear combinations of the input variables are selected. Optimum neural network architectures are organized by using selected useful combinations of the input variables. The logistic GMDH-type neural networks are shown in Fig.3. Here, nonlinear function $g_i$ is described by the following Kolmogorov-Gabor polynomial:

$$g_i\ (x_1,x_2,\cdots,x_p) = a_0+\sum_i a_ix_i+\sum_i\sum_j a_{ij}x_ix_j+\cdots \quad (15)$$

This nonlinear function is automatically organized by using the second type neuron of the conventional GMDH-type neural networks. The architectures of the logistic GMDH-type neural networks are produced as follows:

In the logistic GMDH-type neural networks, the original data need not be separated into training and test sets because the AIC can be used as the test errors. First, the architecture of the first layer is organized.

**1) The first layer**

$$u_j=x_j \quad (j=1,2,\cdots,p) \quad (16)$$

where $x_j$ (j=1,2,$\cdots$,p) are the input variables of the system, and p is the number of input variables. In the first layer, input variables are set to the output variables.

**2) The second layer**

Many combinations of two variables ($u_i$,$u_j$) are generated. For each combination, the neuron architecture is described by the following equations:

$\Sigma$: (Nonlinear function)

$$z_k = w_1u_i+w_2u_j+w_3u_iu_j+w_4u_i^2+w_5u_j^2+w_6u_i^3+w_7u_i^2u_j+w_8u_iu_j^2+w_9u_j^3 - w_0\theta_1 \quad (17)$$

f : (Linear function)

$$y_k = z_k \quad (18)$$

where $\theta_1$ =1 and $w_i$ ( i=0,1,2,$\cdots$,9) are weights between the first and second layer. This neuron is equal to the second type neuron of the conventional GMDH-type neural networks. The weights $w_i$ (i=0,1,2,$\cdots$,9) are estimated by using the logistic regression analysis. This procedure is as follows:

First, the values of $z_k$ are calculated by using the following equation:

$$z_k=log_e(\phi'/(1-\phi')) \quad (19)$$

where $\phi$' is the normalized output variable. Then the weights $w_i$ (i=0,1,2,$\cdots$,9) are estimated by using the stepwise regression analysis [14] which selects useful input variables by using the AIC. Therefore, only useful terms in (17) are selected and neuron architecture can be organized by these selected useful terms.

From these generated neurons, L neurons which minimize the AIC are selected. The output values ($y_k$) of L selected neurons are set to the input values of the neurons in the third layer.

**3) The third layer**

In the third layer and above, the same computation of the second layer is continued until the AIC values of L neurons become identical. When the iterative computation is terminated, the complete neural network architecture is produced by selected neurons in each layer. In the output layer, the output values of the neural networks are calculated from $z_k$ as follows:

$$\phi=1/(1+exp(-z_k)) \quad (20)$$

So, in the output layer, the neuron architecture becomes as follows:

Σ: (Nonlinear function)

$$z_k= w_1u_i+w_2u_j+w_3u_iu_j+w_4u_i^2+w_5u_j^2+w_6u_i^3+w_7u_i^2u_j +w_8u_iu_j^2+w_9u_j^3 - w_0\theta_1 \quad (21)$$

f : (Nonlinear function)

$$\phi=1/(1+exp(-z_k)) \quad (22)$$

This neuron architecture is the same of the first type neuron of the conventional GMDH-type neural networks.

By using above procedures, the logistic GMDH-type neural networks can be constructed. Fig.4 shows the flowchart of the logistic GMDH-type neural networks. In above mentioned logistic GMDH-type neural networks, the first and second type neuron of the conventional GMDH-type neural networks are used and their architectures are automatically selected so as to minimize the AIC. Furthermore, we can also use the fourth type neuron in the hidden layers and the third type neuron in the output layer. When we use these neurons in the logistic GMDH-type neural networks, we can identify the nonlinear system more accurately.

## 4. IDENTIFICATION OF THE X-RAY FILM CHARACTERISTIC CURVE BY USING THE LOGISTIC GMDH-TYPE NEURAL NETWORKS.

The X-ray film characteristic curve of the intensifying screen and film system has been identified by using the logistic GMDH-type neural networks. In this study, the data of the photographic density (D) and the photographic log relative exposure (log RE) which were measured by using the intensity scale method were used for identifying the X-ray film characteristic curve. Fig.5 shows the measurement system of the intensity scale method. The number of exposure points was twenty-one. In this study, the original data were separated into training and test sets so as to check the overfitting of the neural networks. But, the logistic GMDH-type neural networks were organized by using only training set.

### 4.1 The identification results of the X-ray film characteristic curve by using the logistic GMDH-type neural networks.

1) The input variables.

The following four input variables $u_i$ (i=1,2,···,4) were used.

$$u_1=x \quad (23)$$

$$u_2=x^2 \quad (24)$$

$$u_3=x^3 \quad (25)$$

$$u_4=x^4 \quad (26)$$

where x was the value of the log relative exposure (log RE). All of four input variables $u_i$ (i=1,2,···,4) were selected as useful input variables and used in organization of the networks.

2) Self-selection of the optimum neurons.

Four neurons were selected in each hidden layer. The optimum neuron architectures were selected so as to minimize the AIC by using stepwise regression analysis [14].

3) The structure of the neural network.

The calculation of the logistic GMDH-type neural networks was terminated at the fifth layer and the complete neural networks were organized by using selected neurons in the five layers.

4) The estimation accuracy.

The estimation accuracy for training data was evaluated by using the following equation:

$$J_1=1/n_1 \sum_{i=1}^{n1} (\phi_i^*-\phi_i)^2 \quad (27)$$

where $\phi_i^*$ was the estimated value of the i-th training point $\phi_i$ and $n_1$ was the number of the training points equal to 17. The estimation accuracy for the test data was evaluated by using the following equation:

$$J_2=1/n_2 \sum_{i=1}^{n2} (\phi_i^*-\phi_i)^2 \quad (28)$$

where $\phi_i^*$ was estimated value of the i-th test point $\phi_i$ and $n_2$ was the number of the test points equal to 4. $J_1$ and $J_2$ are shown in Table 1.

5) Variation of the AIC.

The variation of the AIC is shown in Fig.6. The variation of the AIC converged in the fifth layer.

6) Estimated values.

The estimated values of the training and test points are shown in Fig.7. We can see that the estimated values are very accurate.

### 4.2 The identification results of X-ray film characteristic curve by using multiple regression analysis, the conventional neural networks and the GMDH method.

In this paper, the identification results obtained by using multiple regression analysis were quoted from [15].

#### 4.2.1 The identification results by using the multiple regression analysis

The polynomial approximation method in the multiple regression analysis was used. As the polynomial, the following linear combination of Legendre's polynomials were used.

$$\phi=C_1P_0(z)+C_2P_1(z)+C_3P_2(z)+\cdots+C_mP_{m-1}(z), \quad (-1 \le z \le 1) \quad (29)$$

where $P_k(z)$ (k=0,1,2,···,m-1) were Legendre's polynomials as follows:

$$P_0(z)=1 \quad (30)$$

$$P_1(z)=z \quad (31)$$

·

·

$$P_k=\{(2k-1)/k\} zP_{k-1}(z) - \{(k-1)/k\} P_{k-2}(z) \quad (32)$$

where

$$z = (2 x - x_{max}- x_{min})/(x_{max}- x_{min}) \quad (33)$$

$x_{max}$ was maximum value and $x_{min}$ was minimum value of the log relative exposure. The parameters $C_k$ (k=1,2,···,m) in (29) were estimated by using the least square estimation method [14]. Table 2 shows the estimation error. The overfitting occurred in terms higher than 9-th order. $J_1$ is shown in Table 1.

#### 4.2.2 The identification results by using the conventional neural networks

The neural networks had three layered structures. Four input variables which were the same of the logistic GMDH-type neural networks were used. Four neurons were used in the hidden layer. All neurons in the neural network were described by the sigmoid functions. The learning of the neural networks was performed using the backpropagation method [8]-[10], repeated 850,000 times. The estimation accuracy for the training data was evaluated by using (27). The estimation accuracy for the test data was evaluated by using (28). $J_1$ and $J_2$ are shown in Table 1.

**4.2.3 The identification results by using the GMDH method**

1) The input variables.

The input variables of the GMDH method were the same of the logistic GMDH-type neural networks. Four variables were used as the input variables and all of them were selected as the useful input variables.

2) Self-selection of the intermediate variables.

Four intermediate variables were selected by using the AIC.

3) The structure of the GMDH.

The GMDH calculation was terminated in the fourth layer and the complete description was organized.

4) The estimation accuracy.

The estimation accuracy for the training data was evaluated by using (27). The estimation accuracy for the test data was evaluated by using (28). $J_1$ and $J_2$ are shown in Table 1.

5) Variation of the AIC.

The variation of the AIC is shown in Fig.8. The variation of the AIC converged in the fourth layer.

**4.3 Comparison of the logistic GMDH-type neural networks and the other models.**

Comparison of the logistic GMDH-type neural networks and the other models, which are the multiple regression analysis, the conventional neural networks and the GMDH method, are shown in Table 1. As seen from Table 1, the estimation accuracy $J_1$ and $J_2$ of the logistic GMDH-type neural networks were smallest in the four identified models and the overhitting didn't occur. In the multiple regression analysis, the overfitting occurred and this model was not accurate and in stable. In the conventional neural networks, the structural identification method has not been developed. So many calculations for various neural network architectures were needed so as to find out desire neural network architectures and the computation time for the learning was very long. In the GMDH method, the estimation accuracy $J_1$ and $J_2$ were greater than those of the logistic GMDH-type neural networks. From this comparison, we can see that the logistic GMDH-type neural networks are very accurate and offer useful identification method for the X-ray film characteristic curve.

## 5. CONCLUSION

In this paper, the logistic GMDH-type neural networks identifying a complex nonlinear system have been proposed. The logistic GMDH-type neural networks are automatically organized by using the heuristic self-organization method. It is very easy to apply this algorithm to the identification problems of the nonlinear system. The logistic GMDH-type neural networks have been applied to the identification problem of the X-ray film characteristic curve. It has been found that the modeling with the logistic GMDH-type neural networks is more accurate than when the multiple regression analysis, the conventional neural networks and the GMDH method are used.

## 6. REFERENCES

[1] A.G.Ivakhnenko, "Heuristic self-organization in problems of engineering cybernetics", *Automatica*, Vol.6, No.2, pp.207-219, 1970.

[2] S.J.Farlow Ed., *Self-organizing Methods in Modeling, GMDH-type Algorithms*, New York , Marcel Dekker, Inc., 1984.

[3] H.R.Madala and A.G.Ivakhnenko, *Inductive Learning Algorithms for Complex Systems Modeling*, Boca Raton, CRC Press, Inc., 1994.

[4] A.G.Ivakhnenko, G.A.lvakhnenko and J.A.Muller, "Self-organization of the neural networks with active neurons", *Pattern Recognition and Image Analysis*, Vol.4, No.2, pp.177-188, 1994.

[5] A.G.Ivakhnenko and J.A.Muller, "Self-organization of nets of active neurons", *SAMS*, Vol.20, pp.93-106, 1995.

[6] H.Tamura and T.Kondo, "Heuristics free group method of data handling algorithm of generating optimum partial polynomials with application to air pollution prediction", *INT. J. SYSTEMS SCI.*, Vol 11, No.9, pp.1095-l11l, 1980.

[7] H.Akaike, "A new look at the statistical model identification", *IEEE Trans. Automatic Control*, Vol.AC-19, No.6, pp.716-723, 1974.

[8] D.T.Pham and X.Liu, *Neural networks for identification, Prediction and Control*, London, Springer-Verlag, 1995.

[9] R.H.Nielsen, *Neurocomputing*, Addison-Wesley, 1990.

[10] J.M.Zurada, *Introduction to Artificial Neural Systems*, Boston, PWS, 1992.

[11] T.Kondo, "GMDH neural network algorithm using the heuristic self-organization method and its application to the pattern identification problem", *Proc. of the 37th SICE Annual Conference, International Session Paper*, pp.1143-1148, 1998.

[12] T.Kondo, "The learning algorithms of the GMDH neural network and their application to the medical image recognition", *Proc. of the 37th SICE Annual Conference, International Session Paper*, pp.1109-1114, 1998.

[13] T.Kondo, "GMDH type Neural Network Algorithm Identifying a Network Structure with the Heuristic Self-Organization Method", *Trans. ISCIE*, Vol.11, No.4, pp.198-207,1998.

[14] N.R.Draper and H.Smith, *Applied Regression Analysis*, New York, John Wiley and Sons, 1981.

[15] T.Kondo, "Identification of the X-ray film characteristic curve by using the neural network", *Bull. Sch. Med. Sci. Univ. Tokushima*, Vol.7, pp9-16, 1997.

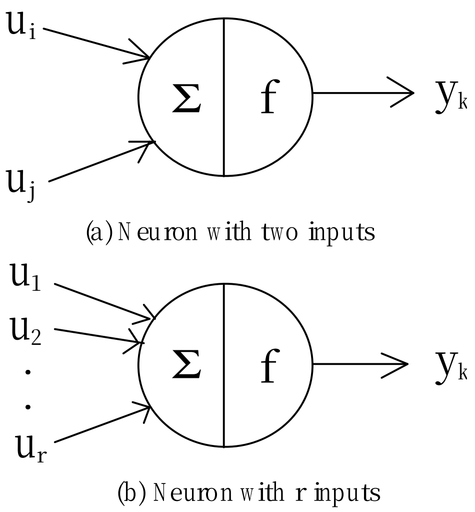
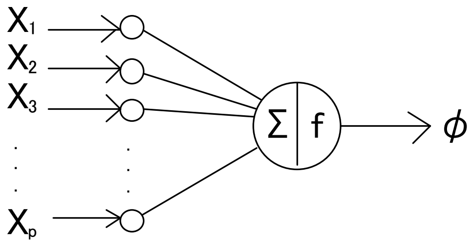



Fig.1 Architectures of the neurons



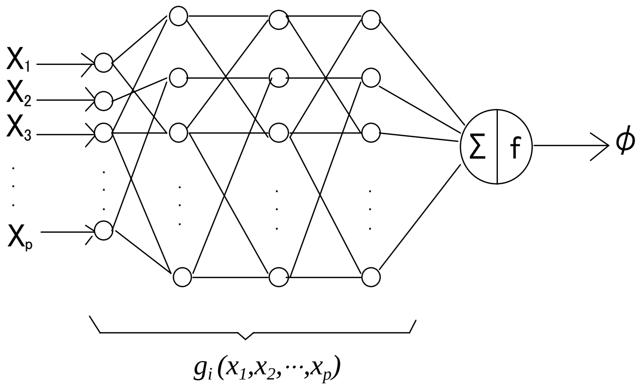

$\Sigma$ *: (linear function)*

$$z = w_1x_1+w_2x_2+\cdots+w_px_p$$

*f : (Nonlinear function)*

$$\phi = 1 / (1+exp(-z))$$

Fig.2 Architecture of the logistic regression analysis




$\Sigma$ *: (Nonlinear function)*

$$z = \sum w_i\, g_i\,(x_1,x_2,\cdots,x_p)$$

*f : (Nonlinear function)*

$$\phi = 1 / (1+exp(-z))$$

Fig.3 Architecture of the logistic GMDH-type neural networks

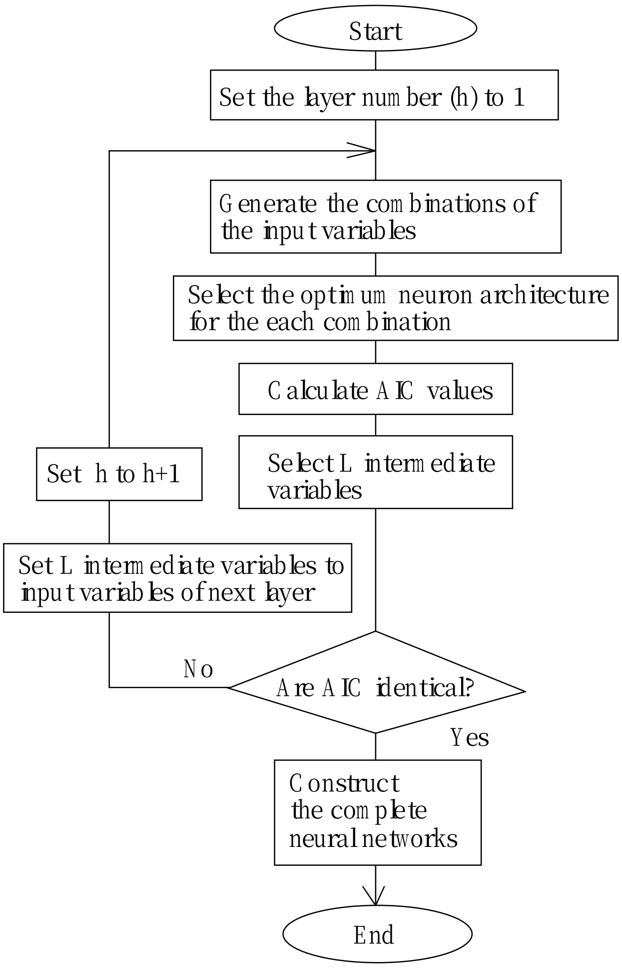



Fig.4 Flowchart of the logistic GMDH-type neural networks.

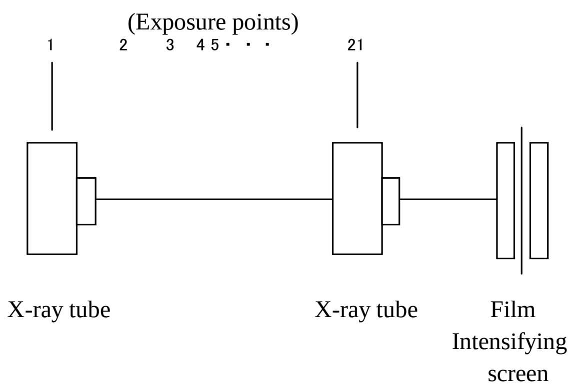



Fig.5 Measurement system of the intensity scale method [15]

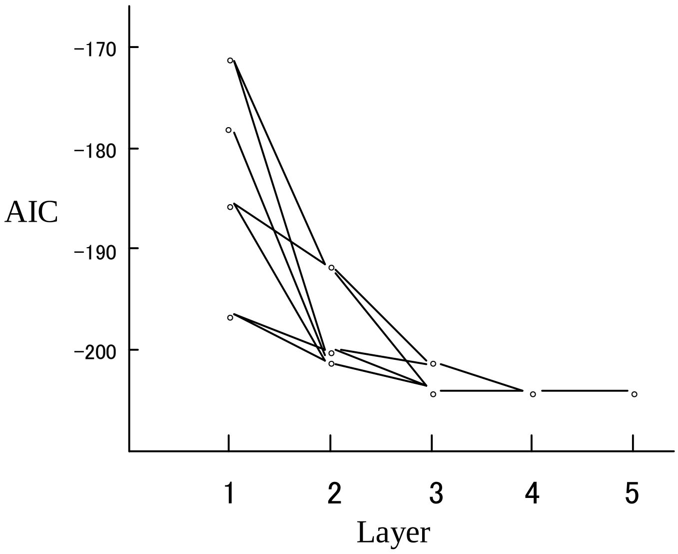



Fig.6 Variation of the AIC by the logistic GMDH-type neural networks

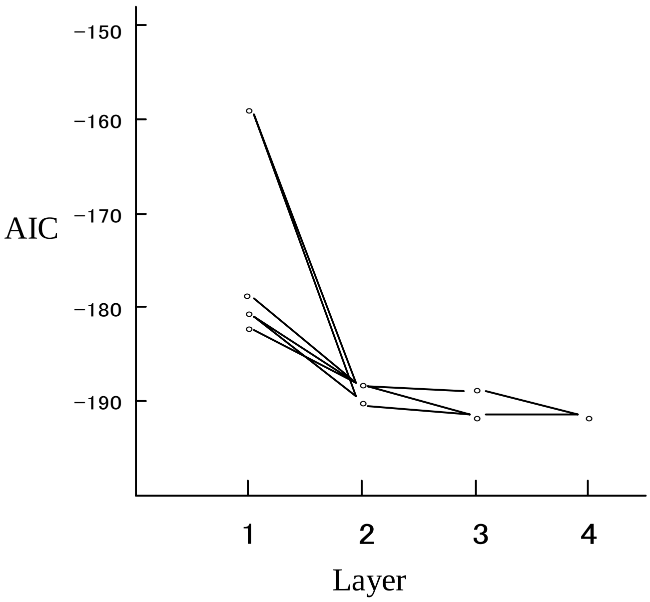



Fig. 8 Variation of the AIC by the GMDH method

Table1 Comparison of four models

| Models | $J_1$ | $J_2$ |
|---|---|---|
| Logistic GMDH-type NN | $6.557\times10^{-6}$ | $1.531\times10^{-5}$ |
| GMDH method | $1.958\times10^{-5}$ | $6.384\times10^{-5}$ |
| Conventional NN | $2.911\times10^{-4}$ | $2.915\times10^{-4}$ |
| Multiple regression analysis | $5.795\times10^{-3}$ | — |

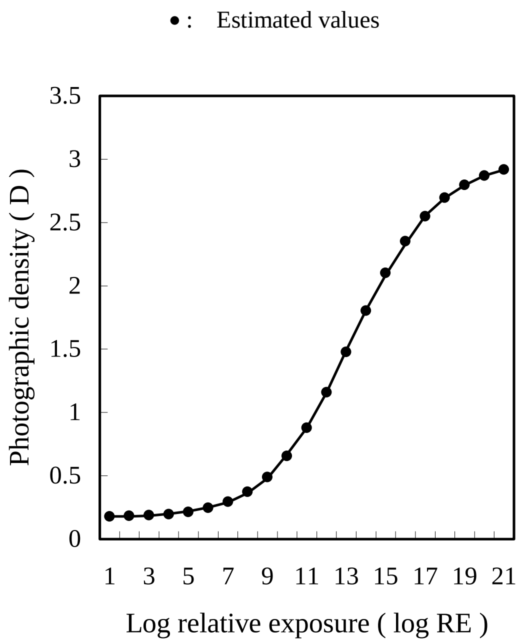



Fig.7 Estimated values by the logistic GMDH-type neural networks

Table 2 Estimation error of the multiple regression analysis [15]

| Order | Estimation error |
|---|---|
| 0 | 1.07621249 |
| 1 | 0.30595454 |
| 2 | 0.21547332 |
| 3 | 0.10865137 |
| 4 | 0.05029438 |
| 5 | 0.03558903 |
| 6 | 0.01319877 |
| 7 | 0.01147893 |
| 8 | 0.00879074 |
| 9 | 0.00892195 |
| 10 | 0.00922519 |
| 11 | 0.00968969 |
| 12 | 0.00916510 |
| 13 | 0.00886184 |
| 14 | 0.00897495 |
| 15 | 0.00579526 |